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REGULAR SUBRING OF A POLYNOMIAL RING

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Introduction. The purpose of this article is to prove the following two theorems:

Theorem 1. *Let k be an algebraically closed field of characteristic zero, and let A be a k-subalgebra of a polynomial ring $B:=k[x, y]$ such that B is a flat A-module of finite type. Then A is a polynomial ring in two variables over k.*

Theorem 2. *Let k be an algebraically closed field of characteristic zero, and let $B:=k[x, y, z]$ be a polynomial ring in three variables over k. Assume that there is given a nontrivial action of the additive group G_a on the affine 3-space $\boldsymbol{A}_k^3:=\mathrm{Spec}(B)$ over k. Let A be the subring of G_a-invariant elements in B. Assume that A is regular. Then A is a polynomial ring in two variables over k.*

Theorem 1 was formerly proved in part under one of the following additional conditions (cf. [7; pp. 139–142]):

(1) B is etale over A,

(2) A is the invariant subring in B with respect to an action of a finite group.

In proofs of both theorems, substantial roles will be played by the following theorem, which is a consequence of the results obtained in Fujita [1], Miyanishi-Sugie [8] and Miyanishi [6]:

Theorem 0. *Let k be an algebraically closed field of characteristic zero, and let $X=\mathrm{Spec}(A)$ be a nonsingular affine surface defined over k. Then the following assertions hold true:*

(1) *X contains a nonempty cylinderlike open set, i.e., there exists a dominant morphism $\rho: X \to C$ from X to a nonsingular curve C whose general fibers are isomorphic to the affine line $\boldsymbol{A}_k^1$, if and only if X has the logarithmic Kodaira dimension $\bar{\kappa}(X)=-\infty$.*

(2) *X is isomorphic to the affiine plane $\boldsymbol{A}_k^2$ if and only if X has the logarithmic Kodaira dimension $\bar{\kappa}(X)=-\infty$, A is a unique factorization domain, and $A^*=k^*$, where A^* is the set of invertible elements in A and $k^*=k-(0)$.*

In this article, the ground field k is always assumed to be an algebraically

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