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A REMARK ON ASYMPTOTIC SUFFICIENCY UP TO HIGHER ORDERS IN MULTI-DIMENSIONAL PARAMETER CASE

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1. Introduction. Suppose that n-dimensional random vector $z_n=(x_1, x_2, \cdots, x_n)$ is distributed according to a probability measure $P_{\theta,n}$ parameterized by$\theta\in\Theta\subset \boldsymbol{R}^p$, and each component x_i is independently and identically distributed. In Suzuki [3] it was shown that when $p=1$ a statistic $t_n^*=(\hat{\theta}_n, \Phi_n^{(1)}(z_n, \hat{\theta}_n), \cdots, \Phi_n^{(k)}(z_n, \hat{\theta}_n))$ is asymptotically sufficient up to order $o(n^{-(k-1)/2})$ in the following sense: For each n t_n^* is sufficient for a family $\{Q_{\theta,n}; \theta\in\Theta\}$ of probability measures and that

$$\lim_{n\to\infty} n^{(k-1)/2}||P_{\theta,n}-Q_{\theta,n}|| = 0$$

uniformly on any compact subset of Θ (where $||\cdot||$ means the total variation norm of a signed measure). Here $\hat{\theta}_n$ is some reasonable estimator of θ and $\Phi_n^{(i)}(z_n, \theta)$ means the i-th logarithmic derivative relative to θ of the density of $P_{\theta,n}$. In this paper we show that the result can be extended to the case where underlying distribution $P_{\theta,n}$ has multi-dimensional parameter θ. Exact form of t_n^* would be found in the statement of the theorem in Section 3. In Michel [2] a similar result was obtained with order of sufficiency $o(n^{-(k-2/)2})$, and hence ours is more accurate one.

2. Notations and assumptions. Let $\Theta(\neq\phi)$ be an open subset of p-dimensional Euclidean space $\boldsymbol{R}^p$. Suppose that for each $\theta\in\Theta$ there corresponds a probability measure P_θ defined on a measurable space $(\boldsymbol{X}, \boldsymbol{A})$. For each $n\in N=\{1, 2, \cdots\}$ let $(\boldsymbol{X}^{(n)}, \boldsymbol{A}^{(n)})$ be the cartesian product of n copies of $(\boldsymbol{X}, \boldsymbol{A})$, and $P_{\theta,n}$ the product measure of n copies of P_θ. For a signed measure λ on $(\boldsymbol{X}^{(n)}, \boldsymbol{A}^{(n)})$, $||\lambda||$ means the total variation norm of λ over $\boldsymbol{A}^{(n)}$. For a function h and a probability P, $E[h; P]$ stands for the expectation of h under P. In the following it will be assumed that the map: $\theta\to P_\theta$ is one to one, and that for each $\theta\in\Theta$ P_θ has a densiy $f(x, \theta)$ relative to a sigma-finite measure μ on $(\boldsymbol{X}, \boldsymbol{A})$. We assume that $f(x, \theta)>0$ for every $x\in \boldsymbol{X}$ and every $\theta\in\Theta$. We denote by μ_n the product measure of n copies of the same com-