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REMARKS ON PROOF OF A THEOREM OF KATO AND KOBAYASI ON LINEAR EVOLUTION EQUATIONS

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1. Introduction

Let

$$du/dt+A(t)u=f(t)\,, \quad 0\leqq t\leqq T\,, \tag{1.1}$$

be an evolution equation of "hyperbolic" type in a Banach space E with $A(t)$ having a domain containing a fixed dense linear subspace F. T. Kato [1], [2], J.R. Dorroh [3], S. Ishii [4],[5], K. Kobayasi [7] etc. have developed methods of constructing an evolution operator for (1.1). The main theorem due to T. Kato and K. Kobayasi is stated as follows:

Theorem. *Let E and F be Banach spaces such that F is densely and continuously embedded in E, and $\{A(t)\}_{0\leqq t\leqq T}$ be a family of closed linear operators in E with the domains*

$$D(A(t))\supset F\,.$$

Assume that

(I) $\{A(t)\}_{0\leqq t\leqq T}$ *is stable on E,*

(II) $A\in C([0, T]; \mathcal{L}_s(F; E))$,

(III) *There is family $\{S(t)\}_{0\leqq t\leqq T}$ of isomorphisms from F onto E such that*

$$S\in C^1([0, T]; \mathcal{L}_s(F; E))\,,$$

and

$$S(t)A(t)S(t)^{-1}=A(t)+B(t)$$

for each $t\in[0, T]$ with some

$$B\in C([0, T]; \mathcal{L}_s(E))\,.$$

Then we can construct an unique evolution operator $\{U(t, s)\}_{0\leqq s\leqq t\leqq T}$ with the following properties

a) $U\in C(\{(t, s); 0\leqq s\leqq t\leqq T\}; \mathcal{L}_s(E))$,

b) $U\in C(\{(t, s); 0\leqq s\leqq t\leqq T\}; \mathcal{L}_s(F))$,

c) $U(t, s)U(s, r)=U(t, r),\ 0\leqq r\leqq s\leqq t\leqq T;\ U(s, s)=I,\ 0\leqq s\leqq T$,