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# MIXED PROBLEMS FOR THE WAVE EQUATION WITH A SINGULAR OBLIQUE DERIVATIVE

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**Introduction.** Let $\Omega$ be a domain in $\boldsymbol{R}^2$ with a compact $C^\infty$ boundary $\Gamma$, and consider the mixed problem

$$(0.1)\qquad \begin{cases} \Box u \equiv \dfrac{\partial^2 u}{\partial t^2} - \Delta_x u = f(x,t) & \text{in } \Omega\times(0,t_0)\,, \\ \dfrac{\partial u}{\partial \nu}\Big|_\Gamma = g(x',t) & \text{on } \Gamma\times(0,t_0)\,, \\ u|_{t=0} = u_0(x) & \text{on } \Omega\,, \\ \dfrac{\partial u}{\partial t}\Big|_{t=0} = u_1(x) & \text{on } \Omega\,, \end{cases}$$

where $\nu=\nu(x)$ is a non-vanishing real $C^\infty$ vector field defined in a neighborhood of $\Gamma$. We say that (0.1) is $C^\infty$ well-posed when there exists a unique solution $u(x,t)$ in $C^\infty(\bar\Omega\times[0,t_0])$ for any $(f,g,u_0,u_1)\in C^\infty(\bar\Omega\times[0,t_0])\times C^\infty(\Gamma\times[0,t_0])\times C^\infty(\bar\Omega)\times C^\infty(\bar\Omega)$ satisfying the compatibility condition of infinite order.

In the case where $\nu$ is not tangent to $\Gamma$ anywhere, various results have been obtained. It has been well known for a long time that the problem (0.1) is $C^\infty$ well-posed if $\nu$ is parallel anywhere to the normal vector $n$ of $\Gamma$ (the Neumann boundary condition). Ikawa [3] showed that (0.1) is $C^\infty$ well-posed also if $\nu$ is oblique (i.e. not parallel to $n$) anywhere on $\Gamma$ (the oblique boundary condition). When these two types are mixed, the shape of $\Omega$ has to be taken into consideration. Ikawa [4, 5, 6] examined it in detail.

In the present paper we shall study (0.1) in the case where $\nu$ is not necessarily non-tangent to $\Gamma$. We assume that $\nu$ is tangent to $\Gamma$ at finite number of points (of $\Gamma$). And we call them singular points. At each singular point the Lopatinski condition is not satisfied; therefore, the mixed problem frozen there is not $C^\infty$ well-posed (cf. Sakamoto [13]). We can classify the behavior of $\nu$ near each singular point into the following three types: As $x'$ ($\in\Gamma$) passes the singular point in the direction of the tangential component of $\nu(x')$ to $\Gamma$,

(I) $\langle \nu(x'), n(x')\rangle$ changes sign from positive to negative;

(II) $\langle \nu(x'), n(x')\rangle$ changes sign from negative to positive;