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ON THE KERNEL OF POSITIVE DEFINITE TYPE

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In a locally compact Hausdorff space, let $k(P, Q)$ be a real-valued function, continuous for any points P and Q, may be ∞ for $P=Q$ and always finite for $P\neq Q$, and $n(P, Q)$ a real-valued function finite and continuous for any points P and Q. A complex-valued function

$$K(P, Q)=k(P, Q)+in(P, Q)$$

is said to be of positive definite type if the double integral (called energy integral)

$$\iint K(P, Q)d\sigma(Q)d\bar{\sigma}(P)$$

of any complex-valued measure σ supported by a relatively compact Borelian set, whenever it is finitely determined, is non-negative. As is well-known, any function $K(P, Q)$ of positive definite type is symmetric:

$$K(P, Q)=\overline{K(Q, P)} \quad i.e. \quad k(P, Q)=k(Q, P) \quad \text{and} \quad n(P, Q)=-n(Q, P),$$

and

$$K(P, P)\geqq 0 \quad \text{and} \quad |K(P, Q)|\leqq \sup K(P, P)$$

for any points P and Q. In the real function theory, we see some results which characterize functions of positive definite type. In the present paper, we shall try to characterize functions of positive definite type on the point of view of the potential theory. We shall advance the argument adopting an idea and a method in the previous paper [2].

For any measure α and β (real-valued or complex-valued) supported by a relatively compact Borelian set, consider the potential taken with respect to a kernel $K(P, Q)$

$$K(P, \alpha)=\int K(P, Q)d\alpha(Q), \qquad K(\alpha, P)=\int K(Q, P)d\alpha(Q)$$

and the double integral (called mutual energy integral)

$$K(\alpha, \beta)=\int d\alpha(P)\int K(P, Q)d\beta(Q).$$