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ON STABILITY OF FINITELY GENERATED KLEINIAN GROUPS

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1. Introduction. The conformal automorphisms of the extended complex plane $\hat{\boldsymbol{C}}=\boldsymbol{C}\cup\{\infty\}$ form the Möbius group $M\ddot{o}b$ Every element α of $M\ddot{o}b$ is a transformation of the form

$$\alpha(z)=(az+b)/(cz+d)\,,$$

where a,b,c and d are complex numbers with $ad-bc=1$. Hence $M\ddot{o}b$ may be considered as a 3-dimensional complex Lie group, isomorphic to $SL(2,\boldsymbol{C})$ modulo its center. We denote by e the identity transformation of $M\ddot{o}b$. An element $\alpha\in M\ddot{o}b$, $\alpha(z)=(az+b)/(cz+d)$, different from e, is called parabolic if $\mathrm{tr}^2\alpha=(a+d)^2=4$; α is called elliptic if $\mathrm{tr}^2\alpha=(a+d)^2\in[0\ 4)$; in all other cases α is called loxodromic.

Let G be a finitely generated Kleinian group, $\Omega=\Omega(G)$ the region of discontinuity of G and $\Lambda=\Lambda(G)$ the limit set of G. Let $M(G)$ be the set of Beltrami coefficients $\mu(z)$ for G supported on $\Omega(G)$, that is, the open unit ball in the closed linear subspace of $L_\infty(\boldsymbol{C})$ determined by the conditions

$$(1.1)\qquad \mu(\gamma z)\overline{\gamma'(z)}/\gamma'(z)=\mu(z),\ (\gamma\in G)$$

and

$$(1.2)\qquad \mu_{|\Lambda(G)}=0\,,$$

where $L_\infty(\boldsymbol{C})$ is the complex Banach space consisting of measurable functions μ on $\boldsymbol{C}$ with finite L_∞ norm $||\mu||$. Let w^μ be the uniquely determined quasiconformal automorphism of $\hat{\boldsymbol{C}}$ with the Beltrami coefficient $\mu=w^\mu_{\bar z}/w^\mu_z$, which keeps the points $0,1,\infty$ fixed. The above condition (1.1) is necessary and sufficient in order that $w^\mu G(w^\mu)^{-1}$ is again a Kleinian group; this is easily checked and is well-known.

Let $\gamma_1,\gamma_2,\cdots,\gamma_k$ be a system of generators for G. A homomorphism χ: $G\rightarrow M\ddot{o}b$ is called parabolic if $\mathrm{tr}^2\chi(\gamma)=4$ for every parabolic element $\gamma\in G$. Let χ: $G\rightarrow M\ddot{o}b$ be a parabolic homomorphism. Then χ is represented by