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# TORSION IN BROWN-PETERSON HOMOLOGY AND HUREWICZ HOMOMORPHISMS

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$BP$ is the Brown-Peterson spectrum (for some prime $p$) and $BP_*X=\pi_*(BP\wedge X)$ is the Brown-Peterson homology of the CW spectrum (or complex) $X$. $BP_*X$ is a left module over the coefficient ring $BP_*\cong Z_{(p)}[v_1, v_2, \cdots]$ and a left comodule over the coalgebra $BP_*BP$. A now classical result is that the stable Hurewicz homomorphism $\pi_*^S X\to H_*(X; Z)$ is an isomorphism modulo torsion. In our context, we restate this as: the Hurewicz homomorphism $h_0(X)\colon \pi_*(BP\wedge X)\to H_*(BP\wedge X; Q)$ has as its kernel the $p$-torsion subgroup of $BP_*X$. This is a prototype of our results.

Instead of restricting our attention to $BP_*X$, it is convenient to study abstract $BP_*BP$-comodules $(M, \psi)$, $\psi\colon M\to BP_*BP\otimes_{BP_*}M$. *A priori*, $M$ is a left $BP_*$-module. As such, it has a richer potential for torsion than mere $p$-torsion. For any polynomial generator $v_n$ of $BP_*$ (by convention $v_0=p$), we say that an element $y\in M$ is $v_n$-torsion if $v_n^s y=0$ for some exponent $s$. If all elements of $M$ are $v_n$-torsion ones, we say that $M$ is a $v_n$-torsion module. If no non-zero element of $M$ is $v_n$-torsion, we say that $M$ is $v_n$-torsion free. Being a $BP_*BP$-comodule severely constrains the $BP_*$-module structure of $M$.

**Theorem 0.1.** *Let $M$ be a $BP_*BP$-comodule. If $y\in M$ is a $v_n$-torsion element, then it is a $v_{n-1}$-torsion element. Consequently, if $M$ is a $v_n$-torsion module, then it is a $v_{n-1}$-torsion module. Or: if $M$ is $v_n$-torsion free, it is $v_{n+1}$-torsion free* (Lemma 2.3 and Proposition 2.5).

The primitive elements of a $BP_*BP$-comodule $M$ are those elements $a$ for which $\psi(a)=1\otimes a$ under $M$'s coproduct $\psi\colon M\to BP_*BP\otimes_{BP_*}M$. We find that some qualitative properties of $BP_*BP$-comodules are determined by these primitives.

**Theorem 0.2** *Let $M$ be an associative $BP_*BP$-comodule. If all the primitives of $M$ are $v_n$-torsion, then $M$ itself is a $v_n$-torsion module. Or: if none of the*

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