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A NOTE ON SULLIVAN COMPLETION

Dedicated to Professor Tatsuji Kudo on his 60th birthday

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In this note we give an alternative construction of the Sullivan finite completion for a "good" space by only making use of the standard techniques in homotopy theory.

Let $c: X \to Y$ be a based map of connected based spaces. We say that c is a π_*-finite completion of X if it is the finite completion on π_1 and π_1-finite completion of the higher homotopy.

Theorem 0 (Sullivan [8, Theorem 3.1. ii), Corollary of proof]).
Let X be a connected based space with "good" homotopy groups. A map $c: X \to Y$ is equivalent to the finite completion if and only if c is a π_*-finite completion.

Sullivan [8; Theorem 3.1. i)] also shows that sufficiently many spaces have "good" homotopy groups. Thus, to construct Sullivan finite completion, it is enough to construct a π_*-finite completion.

Since our arguments are quite formal, analogous l-finite construction is also available for a set l of primes.

1. π_*-finite completion and Main Theorem

Let X be a connected based space and let $\{M_i\}_{i \in I}$ be a projective system of finite $\pi_1 X$-modules and $\pi_1 X$-(equivariant) homomorphisms. Then we have a projective system $\{H^n(X, *; M_i)\}_{i \in I}$ and compatible homomorphisms $H^n(X, *; \lim M_i) \to H^n(X, *; M_i)$, where $H^n(X, *; M_i)$, $H^n(X, *; \lim M_i)$ are *n-th* cohomology groups with twisted coefficients M_i, $\lim M_i$ respectively.

Theorem 1.1. *We have a natural isomorphism*

$$H^n(X, *; \lim M_i) \cong \lim H^n(X, *; M_i).$$

Following [8], we say that π is a good group (resp. a weakly good group) if

$$H^n(\pi; M) \cong \operatorname{colim} H^n(\pi_\alpha; M) \cong H^n(\hat{\pi}; M)$$
$$(\text{resp. } H^n(\pi; M) \cong H^n(\hat{\pi}; M))$$