

ISOTROPIC SUBMANIFOLDS WITH PARALLEL SECOND FUNDAMENTAL FORMS IN SYMMETRIC SPACES

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Introduction. In the study of submanifolds of a riemannian manifold, the notion of an isotropic submanifold has been introduced by B. O'Neill [13] as a generalization of a totally geodesic submanifold. On the other hand, as another generalization of a totally geodesic submanifold, there is the notion of a submanifold with parallel second fundamental form. It is interesting to study submanifolds that belong to both classes, in particular, those which are not totally geodesic, that is, to study nonzero isotropic submanifolds with parallel second fundamental forms. These submanifolds have the property that every geodesic in the submanifold is a circle in the ambient riemannian manifold (K. Nomizu [11]).

Now, as typical examples of such submanifolds, we have the following two; an extrinsic sphere and a nonzero isotropic Kähler submanifold with parallel second fundamental form. The former submanifold is totally umbilical and the latter is minimal.

When the ambient riemannian manifold is a symmetric space, extrinsic spheres have been studied by B.Y. Chen ([2],[3],[4]). Moreover when the ambient riemannian manifold is a complex projective space with Fubini-Study metric, nonzero isotropic Kähler submanifolds with parallel second fundamental forms have been studied by K. Nomizu [11] and T. Itoh [8].

In this paper, we shall show the following two results:

I) If the ambient riemannian manifold is a symmetric space, a complete extrinsic sphere of dimension ≥ 2 is isometric to a simply connected real space form (Theorem 8).

II) If the ambient riemannian manifold is a Hermitian symmetric space, a complete nonzero isotropic Kähler submanifold with parallel second fundamental form is the Veronese submanifold of degree 2 in some totally geodesic complex projective space in the Hermitian symmetric space (Theorem 25).

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