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# ON THE SPECTRUM OF A RIEMANNIAN MANIFOLD OF POSITIVE CONSTANT CURVATURE

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**Introduction.** Let $M$ be a compact connected riemannian manifold and $\Delta$ the Laplacian acting on the space of $C^\infty$-functions on $M$. The operator $\Delta$ has a discrete spectrum consisting nonnegative eigenvalues with finite multiplicities. We denote by Spec $M$ the spectrum of the $\Delta$. Two compact connected riemannian manifolds $M$ and $N$ are said to be *isospectral* to each other if Spec $M=$Spec $N$. The spectrum of a riemannian manifold gives a lot of information about its riemannian structure, but it does not completely determine the riemannian structure in general. In fact, there exist two flat tori which are isospectral but not isometric (by J. Milnor, see [1]). On the other hand some distinguished riemannian manifolds are completely characterized by their spectra as riemannian manifolds. The $n$-dimensional sphere $S^n$ with the canonical metric and the real projective space $P^n(\boldsymbol{R})$ with the canonical metric are completely characterized by their spectra as riemannian manifolds if $n\leqq 6$ (see [1], [8]). Recently, it has been shown successively that a 3-dimensional lens space $M$ is completely determined by its spectrum as a riemannian manifold; first by M. Tanaka [7] in the case the order $|\pi_1(M)|$ of the fundamental group of $M$ is odd prime or 2-times odd prime, then by the author and Y. Yamamoto [4] in a more general case, and finally by Y. Yamamoto [11] without any restriction. These examples are riemannian manifolds of positive constant curvature.

*A connected complete riemannian manifold* $M$ ($\dim M\geqq 2$) *of positive constant curvature* 1 *is called a spherical space form.* Now, we consider the problem;

(0.1) *Is a spherical space form characterized by its spectrum among all spherical space forms ?*

In this paper, we shall adapt one method to solve the problem and show affirmative results for the problem in the cases where spherical space forms are 3-dimensional and where spherical space forms are homogeneous.

Let $S^n$ ($n\geqq 2$) be the $n$-dimensional sphere of constant curvature 1. If $M$ is an $n$-dimensional spherical space form, then there exists a finite group $G$ of fixed point free isometries on $S^n$ such that $M$ is isometric to $S^n/G$. Let $E_k$ ($k\geqq 0$)