Koiso, N.
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RIGIDITY AND STABILITY OF EINSTEIN METRICS —THE CASE OF COMPACT SYMMETRIC SPACES

NORIHITO KOISO

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1. Introduction and results

Let M be a compact connected manifold of $\dim M \geqq 2$ and g an Einstein metric on M. If (M, g) is the standard sphere, then all Einstein metrics g' on M near g are of constant sectional curvature, and so (M, g') are homothetic with (M, g) (Berger [2] Proposition 6.4, Muto [23] p457 Theorem). Such an Einstein metric g is said to be *rigid*. We know that some of Einstein metrics with vanishing Ricci tensors are not rigid. For example, flat torus and the $K3$-surfaces are not rigid (Bourguignon [6] 08). But we know few Einstein metrics with negative definite Ricci tensors which are not rigid. In fact, in this paper we prove the rigidity of Einstein metrics g such that the universal riemannian covering manifold of (M, g) is a symmetric space of non-compact type without 2-dimensional factors (Corollary 3.4). Furthermore, for irreducible locally symmetric spaces of compact type, we show the following.

Theorem 1.1. *The following simply connected symmetric spaces are infinitesimally deformable.* (*For the definition of the infinitesimal deformability, see Definition* 2.4.)

$SU(n+1)$ $(n \geqq 2)$, $SU(n)/SO(n)$ $(n \geqq 3)$, $SU(2n)/Sp(n)$ $(n \geqq 3)$, E_6/F_4.

Theorem 1.2. *Let* (M, g) *be an irreducible locally symmetric space of compact type. If the universal riemannian covering manifold of* (M, g) *is neither one of the types in Theorem* 1.1 *nor of the type* $U(p+q)/U(p) \times U(q)$ $(p \geqq q \geqq 2)$, *then* g *is rigid.*

Moreover we study the stability of Einstein metrics. It is well-known that Einstein metrics g are nothing but critical metrics with respect to the total scalar curvature T (Hilbert [12]). In general, this critical point is neither maximal nor minimal (Berger [1] p290, Muto [24] p 521 Theorem). But if we consider only metrics of constant scalar curvature, then some critical points are maximal. That is, if we denote by C the set of all riemannian metrics on M of constant scalar curvature and with volume 1, then some Einstein metrics