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STRONGLY SEMIPRIME RINGS AND NONSINGULAR QUASI-INJECTIVE MODULES

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Following Handelman [8] we call a ring R is a right strongly semiprime ring provided if I is a two-sided ideal of R and is essential as a right ideal, then it contains a finite subset whose right annihilator is zero.

In this paper, we first show that a ring R is a right strongly semiprime ring if and only if

(1) $Q(R)$ is a direct sum of simple rings, and

(2) $eQ(R)eR=eQ(R)$ for all idempotents e in $Q(R)$ where $Q(R)$ denotes the maximal ring of right quotients of R.

Using these conditions (1) and (2), we shall investigate the following conditions:

(a) Every nonsingular quasi-injective right R-module is injective.

(b) Any finite direct sum of nonsingular quasi-injective right R-modules is quasi-injective.

(c) Any direct sum of nonsingular quasi-injective right R-modules is quasi-injective.

(d) Any direct product of nonsingular quasi-injective right R-modules is quasi-injective.

It is shown that the conditions (a), (b) and (d) are equivalent; indeed, the rings satisfying one of these conditions are determined as rings R such that $R/G(R)$ is a right strongly semiprime ring, where $G(R)$ denotes the right Goldie torsion submodule of R. A ring R satisfying the condition (c) is also characterized as a ring R such that $R/G(R)$ is a semiprime right Goldie ring.

1. Preliminaries and notations

Throughout this paper all rings considered have identity and all modules are unitary.

Let R be a ring. $Q(R)$ denotes its maximal ring of right quotients. Let M be a right R-module. By $E_R(M)$, nM, $Z(M)$ and $G(M)$ we denotes its injective hull, the direct product of n-copies, its singular submodule and its Goldie torsion submodule, respectively. (Note that $Z(M/Z(M))=G(M)/Z(M)$.) For