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OVERRINGS OF KRULL ORDERS

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Introduction. Recently, one of the authors introduced a Krull order R in a simple artinian ring Q[6], that is, R is called Krull if the following conditions hold:

(K1) $R=\cap_{i\in I}R_i\cap S(R)$, where R_i and $S(R)$ are essential overrings of R(see [6] for the definition), and $S(R)$ is the Asano overring of R;

(K2) each R_i is a noetherian, local, Asano order in Q, and $S(R)$ is a noetherian, simple ring;

(K3) if c is any regular element of R, then $cR_i\neq R_i$ for only finitely many i in I and $R_kc\neq R_k$ for only finitely many k in I.

The fundamental properties of Krull orders were studied in [6]. Let $\boldsymbol{P}$ be the set of all prime v-ideals of R and $\boldsymbol{P}_0$ any subset of $\boldsymbol{P}$. Then, in §1, we shall show that an order $T=\cap_{P\in\boldsymbol{P}_0}R_P\cap S(R)$ is also Krull and, in particular, T is an RI-order in the sense of Cozzens and Sandomierski [1], if we take $\boldsymbol{P}_0$ to be the set of all invertible prime ideals of R. In §2 we apply the results of §1 to the case where R is a D-order in a central simple algebra, where D is a unique factorization domain. §3 is devoted to state an example of a maximal order which has the noninvertible prime v-ideals.

1. Overrings of Krull orders. Let R be an order in a simple artinian ring Q. A right R-submodule X of Q is called a *right R-ideal,* if $aR\supset X\supset bR$ for units a,b in Q. A *left R-ideal* and a *two-sided R-ideal* are defined by the similar way. An R-ideal in R is simply called an ideal. For a one-sided R-ideal X of R, put $O_r(X)=\{x\in Q;\ Xx\subset X\}$, $O_l(X)=\{x\in Q;\ xX\subset X\}$, $X^{-1}=\{x\in Q;\ XxX\subset X\}=\{x\in Q;\ Xx\subset O_l(X)\}=\{x\in Q;\ xX\subset O_r(X)\}$, and $X^*=X^{-1-1}$. X is called a *v-ideal* (*invertible ideal*), if $X=X^*(R=XX^{-1}=X^{-1}X)$.

We state some results in [6] concerning Krull orders. Let R be a Krull order in a simple artinian ring Q. R is a maximal order [6, Proposition 2.1]. Let P_i' be a unique maximal ideal of R_i. Then $P_i=P_i'\cap R$ is a prime v-ideal of R(cf. [4, Lemma 1.5]), $P_i'=R_iP_i$ [3, Proposition 1.1], and $R_i=R_{P_i}$, where R_{P_i} is the localization of R at P_i, that is, $R_{P_i}=\{xy^{-1}\in Q;\ x\in R, y\in C(P_i)\}$ with $C(P_i)=\{y\in R; y+P_i$ is a regular element of $R/P_i\}$.

Let $\boldsymbol{P}=\{P_i; i\in I\}$ be the set of all prime v-ideals of R(*cf.* [4, Proposition