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DOUBLY TRANSITIVE GROUPS OF EVEN DEGREE WHOSE ONE POINT STABILIZER HAS A NORMAL SUBGROUP ISOMORPHIC TO $PSL(3,2^n)$

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1. Introduction

Let G be a doubly transitive permutation group on a finite set Ω and $\alpha\in\Omega$. By [4], the product of all minimal normal subgroups of G_α is the direct product $A\times N$, where A is an abelian group and N is 1 or a nonabelian simple group.

In this paper we consider the case $N\simeq PSL(3,q)$ with q even and prove the following:

Theorem. *Let G be a doubly transitive permutation group on Ω of even degree and let α, $\beta\in\Omega$ $(\alpha\neq\beta)$. If G_α has a normal subgroup N^α isomorphic to $PSL(3,q)$, $q=2^n$, then N^α is transitive on $\Omega-\{\alpha\}$ and one of the following holds:*

(i) *G has a regular normal subgroup E of order $q^3=2^{3n}$, where n is odd and G_α is isomorphic to a subgroup of $\Gamma L(3,q)$. Moreover there exists an element g in $Sym(\Omega)$ such that $\alpha^g=\alpha$, $(G_\alpha)^g$ normalizes E and $A\Gamma L(3,q)\geq(G_\alpha)^gE\geq ASL(3,q)$ in their natural doubly transitive permutation representation.*

(ii) $|\Omega|=22$, $G^\Omega=M_{22}$ and $N^\alpha\simeq PSL(3,4)$.

(iii) *$|\Omega|=22$, $G^\Omega=Aut(M_{22})$ and $N^\alpha\simeq PSL(3,4)$.*

We introduce some notations.

$V(n,q)$	:	a vector space of dimension n over $GF(q)$
$\Gamma L(n,q)$	:	the group of all semilinear automorphism of $V(n,q)$
$A\Gamma L(n,q)$	:	the semidirect product of $V(n,q)$ by $\Gamma L(n,q)$ in its natural action
$ASL(n,q)$	:	the semidirect product of $V(n,q)$ by $SL(n,q)$ in its natural action
$F(X)$	:	the set of fixed points of a nonempty subset X of G
$X(\Delta)$	:	the global stabilizer of a subset Δ $(\subseteq\Omega)$ in X
X_Δ	:	the pointwise stabilizer of Δ in X
X^Δ	:	the restriction of X on Δ
Sym(Δ)	:	the symmetric group on Δ