Hiramine, Y.
Osaka J. Math.
16 (1979), 797-816

ON SOME DOUBLY TRANSITIVE PERMUTATION GROUPS IN WHICH SOCLE(G_α) IS NONSOLVABLE

YUTAKA HIRAMINE

(Received July 3, 1978)

1. Introduction

Let G be a doubly transitive permutation group on a finite set Ω and $\alpha \in \Omega$. In [8], O'Nan has proved that socle $(G_\alpha)=A\times N$, where A is an abelian group and N is 1 or a nonabelian simple group. Here socle (G_α) is the product of all minimal normal subgroups of G_α.

In the previous paper [4], we have studied doubly transitive permutation groups in which N is isomorphic to $PSL(2,q)$, $Sz(q)$ or $PSU(3,q)$ with q even. In this paper we shall prove the following:

Theorem. *Let G be a doubly transitive permutation group on a finite set Ω with $|\Omega|$ even and let $\alpha \in \Omega$. If G_α has a normal simple subgroup N^α isomorphic to $PSL(2,q)$, where q is odd, then one of the following holds.*

(i) *G^Ω has a regular normal subgroup.*

(ii) *$G^\Omega \simeq A_6$ or S_6, $N^\alpha \simeq PSL(2,5)$ and $|\Omega|=6$.*

(iii) *$G^\Omega \simeq M_{11}$, $N^\alpha \simeq PSL(2,11)$ and $|\Omega|=12$.*

In the case that G^Ω has a regular normal subgroup, by a result of Hering [3] we have $(|\Omega|, q)=(16,9)$, $(16,5)$ or $(8,7)$.

We introduce some notations:

$F(X)$: the set of fixed points of a nonempty subset X of G
$X(\Delta)$: the global stabilizer of a subset $\Delta(\subseteq\Omega)$ in X
X_Δ : the pointwise stabilizer of Δ in X
X^Δ : the restriction of X on Δ
$m|n$: an integer m divides an integer n
X^H : the set of H-conjugates of X
$|X|_p$: maximal power of p dividing the order of X
$I(X)$: the set of involutions in X
D_m : dihedral group of order m

In this paper all sets and groups are finite.