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STANDARD COMPONENTS OF TYPE M_{12} AND $\cdot 3$

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Intensive activity in the course of the past few years has brought very close to completion the following problem.

PROBLEM. Let G be a finite group with $F^*(G)$ simple. Let T be a subgroup of G and L a subnormal subgroup of $C_G(T)$ with $L/O(L)$ isomorphic to a known quasisimple group. Identify G.

The main contribution to the solution of this problem is the Unbalanced Group Theorem, whose proof now appears to be nearing completion.

Theorem 1.1 (Unbalanced group theorem). *Let G be a finite group with $F^*(G)$ simple. Let t be an involution of G. Then either G is known or* $0(C_G(t))=1$.

We shall call a group G balanced if $0(C_G(t))\subseteq 0(G)$ for all involutions t of G. A crucial corollary to the unbalanced group theorem is the $B(G)$ theorem. Before stating this result, we must review some definitions. A perfect subnormal subgroup L of H is said to be a 2-component if $L/0(L)$ is quasisimple. We say that L is a component if $0(L)\subseteq Z(L)$. The 2-layer of H, denoted $L_{2'}(H)$ is the product of all 2-components of H. Similarly, the layer of H, denoted $L(H)$, is the product of all components of H.

Theorem 1.2 ($B(G)$ theorem). *Let G be a finite group with* $0(G)=1$. *Let t be an involution of L. Then every 2-component of $C_G(t)$ is a component of $C_G(t)$.*

The next major contribution to our problem is the Component theorem of Aschbacher and Foote. For G a finite group, let $\mathcal{L}(G)$ be the set of all components of $C_G(t)$ for t ranging over the involutions of G. We define a relation $<$ on $\mathcal{L}(G)$ as follows:

$K<L$ if there exists a pair (s,t) of commuting involutions with K a component of $C_G(s)$, L a component of $C_G(t)$ and $K\subseteq LL^s$.

We extend $<$ to a transitive relation $\ll$ on $\mathcal{L}(G)$. We say that K is maximal in $\mathcal{L}(G)$ if $K\ll L$ implies $K\cong L$. Finally we say that K is standard in

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