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ON LOGARITHMIC $K3$ SURFACES

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Introduction. By surfaces we mean non-singular algebraic surfaces defined over the field of complex numbers $\boldsymbol{C}$. *A logarithmic* $K3$ *surface* S is by definition a surface S with $\bar{p}_g(S)=1$, $\bar{\kappa}(S)=\bar{q}(S)=0$, in which $\bar{p}_g(S)$ is the *logarithmic geometric genus*, $\bar{\kappa}(S)$ is the *logarithmic Kodaira dimension*, and $\bar{q}(S)$ is the *logarithmic irregularity*. These notions will be explained in §1.

Let $\bar{S}$ be a completion of S with *ordinary boundary* D, *i.e.*, $\bar{S}$ is a non-singular complete surface and D is a divisor with normal crossings on $\bar{S}$ such that $S=\bar{S}-D$. We write D as a sum of irreducible components: $D=C_1+\cdots+C_s$.

Logarithmic $K3$ surfaces are classified into the following three types:
Type I) $p_g(\bar{S})=1$; Then $\bar{S}$ is a $K3$ surface and D consists of non-singular rational curves C_i with negative-definite intersection matrix $[(C_i, C_j)]$.
Type $\mathrm{II_a}$) $p_g(\bar{S})=0$ and a component C_1 of D is a non-singular elliptic curve; Then $\bar{S}$ is a rational surface and the graph of D has no cycles.
Type $\mathrm{II_b}$) $p_g(\bar{S})=0$ and D consists of rational curves C_j; Then $\bar{S}$ is a rational surface and the graph of D has one cycle.

We define *A-boundary* D_A and *B-boundary* D_B of $(\bar{S}, D)$ as follows: 1) If S is of type I, then $D_A=\phi$ and $D_B=D$. 2) If S is of type $\mathrm{II_a}$, then $D_A=C_1$ (a non-singular elliptic curve) and $D_B=C_2+\cdots+C_s$. 3) If S is of type $\mathrm{II_b}$, then $D_A=C_1+\cdots+C_r$ that is a *circular boundary* (for definition, see §1 v)) and $D_B=C_{r+1}+\cdots+C_s$.

Theorem 1. *If* $\bar{S}-D_A$ *has no exceptional curves of the first kind, then* $K(\bar{S})+D_A\sim 0$.

Next, consider the case where $\bar{S}-D_A$ has exceptional curves. Let ρ: $\bar{S}\rightarrow\bar{S}_*$ be a contraction of exceptional curves of the first kind on $\bar{S}-D_A$, *i.e.*, $\bar{S}_*$ is a complete surface and ρ is biregular around D_A such that $\bar{S}_*-\rho(D_A)$ has no exceptional curves of the first kind. By Theorem 1, $K(\bar{S}_*)+\rho(D_A)\sim 0$.

Theorem 2. $\rho(D_B)$ *is a divisor with simple normal crossings. Let* $\mathcal{Z}_1, \cdots, \mathcal{Z}_u$ *be the connected components of* $\rho(D_B)$. *Then* 1) *if* $\mathcal{Z}_i\cap\rho(D_A)\neq\phi$, $\mathcal{Z}_i$ *is an exceptional curve of the first kind such that* $(\mathcal{Z}_i, \rho(D_A))=1$. 2) *If* $\mathcal{Z}_i\cap\rho(D_A)=\phi$,