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ON THE INTERMEDIATE COHOMOLOGY GROUP OF A HOLOMORPHIC LINE BUNDLE OVER A COMPLEX TORUS

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Let $E=V/L$ be a complex torus, where V is an n-dimensional complex vector space and L a lattice of V. Let H be a Hermitian form on V and A the imaginary part of H. Then A is an $\boldsymbol{R}$-bilinear alternating form on V. We say that H is *a Riemann form of signature* (s, r) for the torus E if

(a) H is non-degenerate and of signature (s, r);
(b) A is integral valued on the lattice L.

To a Riemann form H we associate a factor

$$J_{H,\psi}(g, z) = \psi(g)\varepsilon\left[\frac{1}{2i}H(z, g)+\frac{1}{4i}H(g, g)\right] \tag{1}$$

with $g \in L$, $z\in V$, where $\varepsilon[\cdot]=\exp 2\pi i\cdot$ and ψ is a map from L to $\boldsymbol{C}_1^*=\{z\in\boldsymbol{C}\,|\,|z|=1\}$ satisfying $\psi(g+h)=\psi(g)\psi(h)\varepsilon\left[\frac{1}{2}A(g, h)\right]$; the function ψ being called a semi-character of L for A.
The factor $J_{H,\psi}$: $L\times V\to\boldsymbol{C}^*$ satisfies the equation

$$J_{H,\psi}(g+f, z) = J_{H,\psi}(g, h+z)J_{H,\psi}(h, z)\,,$$

where $g, h\in L$, $z\in V$.

Given the factor $J_{H,\psi}$ we define an action of the lattice group L on $V\times\boldsymbol{C}$ by the rule

$$(z, \xi)\cdot g = (z+g, J_{H,\psi}(g, z)\xi)\,,$$

where $z\in V$, $\xi\in\boldsymbol{C}$ and $g\in L$. The action of L on $V\times\boldsymbol{C}$ is free and the quotient of $V\times\boldsymbol{C}$ by L has a natural structure of a holomorphic line bundle over the complex torus $E=V/L$. We shall denote this line bundle by $F(H, \psi)$.

The following vanishing theorem for the cohomology of $F(H, \psi)$ is well-known [2, 4]: If H is a Riemann form of signature (s, r), then we have

$$H^q(E, F(H, \psi)) = 0$$