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Osaka J. Math.
16 (1979), 581–588

ON J_R-HOMOMORPHISMS

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(Received October 13, 1978)

1. Introduction

In [8] Snaith proved the Adams conjecture for suspension spaces. In this paper we shall prove an analogous result to Snaith's theorem ([8], Corollary 5.2) for the Real Adams operation ψ^3 and a Real J-map J_R (see §2). This is proved by using the results of Seymour [7]. And as an application we shall determine an undecided order in the theorem of [6].

Here we shall inherit the notations and terminologies in [2], §1 and [6].

2. Homomorphism J_R

In [6] we defined the homomorphisms $J_{R,n}$ and J_R for doubly indexed suspension spaces $\Sigma^{p,q}X$, $p\geqq 0$ and $q\geqq 1$. Clearly, these definitions are also valid for any finite pointed τ-complex. But the natural map obtained in this manner

$$J_R\colon \widetilde{KR}^{-1}(X)\rightarrow \pi_s^{0,0}(X)$$

is not a homomorphism in general. As in the usual case we see that this map satisfies the following formula:

$$J_R(\alpha+\beta)=J_R(\alpha)+J_R(\beta)+J_R(\alpha)J_R(\beta) \qquad \alpha,\beta\in\widetilde{KR}^{-1}(X)$$

where ab ($a,b\in\pi_s^{0,0}(X)$) denotes the product of a and b induced by the loop composition in $\Omega^{n,n}\Sigma^{n,n}$ (cf. [9],p. 314).

3. Adams operation ψ^3 in KR-theory

In this section we recall the construction of the Real Adams operation ψ_R^3 described in [7], §4.

Let S_3 be the symmetric group with two generators a, b satisfying

$$a^3=b^2=1,\ bab=a^2$$

and let Z_3 be the cyclic subgroup of S_3 generated by a. From the above relations we see that $\tau(a)=a^2$, $\tau(b)=b$ induces an automorphic involution τ on