Uchida, F.
Osaka J. Math.
16 (1979), 561-579

CLASSIFICATION OF REAL ANALYTIC $SL(n, R)$ ACTIONS ON n-SPHERE

Dedicated to Professor A. Komatu on his 70th birthday

FUICHI UCHIDA*)

(Received March 17, 1978)

0. Introduction

C.R. Schneider [5] classified real analytic $\boldsymbol{SL}(2, \boldsymbol{R})$ actions on closed surfaces. Except for the work, there seems to be no work on the classification problem about non-compact Lie group actions.

In this paper, we classify real analytic $\boldsymbol{SL}(n, \boldsymbol{R})$ actions on the standard n-sphere for each $n \geqslant 3$. Here $\boldsymbol{SL}(n, \boldsymbol{R})$ denotes the special linear group over the field of real numbers. The result can be stated roughly as follows: there is a one-to-one correspondence between real analytic $\boldsymbol{SL}(n, \boldsymbol{R})$ actions on the n-sphere and real valued real analytic functions on an interval satisfying certain conditions (see Theorem 2.2 and Theorem 4.2). It is important to consider the restricted actions of $\boldsymbol{SL}(n, \boldsymbol{R})$ to a maximal compact subgroup $\boldsymbol{SO}(n)$.

It is still open to classify C^∞ actions of $\boldsymbol{SL}(n, \boldsymbol{R})$ on the standard n-sphere, by lack of C^∞ analogue of a local theory due to Guillemin and Sternberg (see Lemma 4.3).

1. Real analytic $SO(n)$ actions on certain n-manifolds

First we prepare the following two lemmas of which proof is given in the last section.

Lemma 1.1. *Let G be a closed connected subgroup of $\boldsymbol{O}(n)$. Suppose that $n \geqslant 3$ and*

$$\dim \boldsymbol{O}(n) > \dim G \geqslant \dim \boldsymbol{O}(n) - n .$$

Suppose that G is not conjugate to $\boldsymbol{SO}(n-1)$ which is canonically imbedded in $\boldsymbol{O}(n)$. Then the pair $(\boldsymbol{O}(n), G)$ is pairwise isomorphic to one of the following:

$$(\boldsymbol{O}(8), \boldsymbol{Spin}(7)), (\boldsymbol{O}(7), \boldsymbol{G}_2), (\boldsymbol{O}(6), \boldsymbol{U}(3)), (\boldsymbol{O}(4), \boldsymbol{U}(2)),$$
$$(\boldsymbol{O}(4), \boldsymbol{SU}(2)), (\boldsymbol{O}(4), \boldsymbol{SO}(2) \times \boldsymbol{SO}(2)) \text{ and } (\boldsymbol{O}(3), \{1\}),$$

*) Supported by Grant-in-Aid for Scientific Research