

## KNOTS OF UNKNOTTING NUMBER 1 AND THEIR ALEXANDER POLYNOMIALS

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In 1934 [3], H. Seifert gave a characterization of the Alexander polynomial of knots, but it has been unknown whether there are some relations between the Alexander polynomial and the unknotting number or not.

In this paper, after establishing normal forms of knots with unknotting number  $\leq n$ , we prove that the Alexander polynomial of any knot is able to be obtained from knots with unknotting number 1. This result shows that there is no relation between the Alexander polynomial and the unknotting number.

### 1. Normal form of a knot of unknotting number 1

Let  $axb$  be an arbitrary arc of the given knot  $k$  and let  $D$  be a disk such that

- (i)  $\dot{D} \cap k = axb$ , and
- (ii)  $\overset{\circ}{D} \cap k = \{d\}$ ,  $d$  is a point of  $k$ ,

where  $\dot{D}$  and  $\overset{\circ}{D}$  denote respectively the boundary and the interior of  $D$ .  $D$  will be called an *unknotting disk*. The operation of exchanging the arc  $axb$  by its complementary arc  $ayb$  with respect to  $\dot{D}$  will then be called an *unknotting operation*.

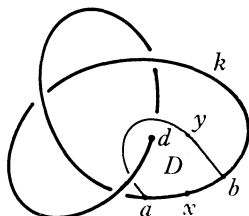


Fig. 1

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