Matsuoka, T. and Morita, S.
Osaka J. Math.
16 (1979), 539-550

ON CHARACTERISTIC CLASSES OF KÄHLER FOLIATIONS

TAKASHI MATSUOKA AND SHIGEYUKI MORITA[1)]

(Received April 28, 1978)

0. Introduction

The purpose of this note is to study Kähler foliations, which are defined by requiring the transition functions to be holomorphic isometries of a Kähler manifold (see Definition 1.1), by adopting the method of [6] [7] [10]. In some sense Kähler foliations are the holomorphic analogue of Riemannian foliations and characteristic classes of the latter have been profoundly investigated by Lazarov and Pasternack (cf.[9], also see [6]). However from the view point of characteristic classes, the situations are completely different. Namely the vanishing phenomenon of the Pontrjagin classes of the normal bundles in the Riemannian case is much stronger than that in the smooth case (cf. strong vanishing theorem of Pasternack [12] and the Bott's vanishing theorem [1]). By contrast, we do not have any strong vanishing phenomenon in the Kähler foliations. This fact reflects in the secondary characteristic classes. For example, all the secondary classes of smooth foliations are zero on Riemannian foliations, but some of the secondary classes of holomorphic foliations may be non-zero on Kähler foliations. A new ingredient of our context is the Kähler form which is a closed 2-form defined for any Kähler foliation.

In § 1 we define Kähler foliations and construct characteristic classes of them and in § 2 we compute the cohomology of certain truncated Weil algebra. In §§ 3 and 4, we study the relationships of our characteristic classes with those of Riemannian and holomorphic foliations. Finally in § 5 we consider deformations of Kähler foliations.

1 Construction of the characteristic classes

In this section we define the notion of Kähler foliations and construct characteristic classes of them.

DEFINITION 1.1. A codimension n Kähler foliation F on a smooth manifold M is a maximal family of submersions $f_\alpha\colon U_\alpha \to (C^n, g_\alpha)$, where U_α is an

1) Partially supported by the Sakkokai Foundation