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ON REAL J-HOMOMORPHISMS

Dedicated to Professor A. Komatu on his 70th birthday

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1. In the present work we consider a Real analogue of J-homomorphisms in the sense of [3]. We use here the notation in [4], §§1 and 9 and [9], §2 for the equivariant homotopy groups which are discussed by Bredon [5] and Levine [10]. Moreover we shall use notations and terminologies of [4], §1 without any references.

Let us denote by $GL(n,\boldsymbol{C})$ (resp. $GL(\infty,\boldsymbol{C})$) the general linear group of degree n (resp. the infinite general linear group) over the complex numbers with involutions induced by complex conjugation. Let X be a finite pointed τ-complex. Then, by following the construction of usual J-homomorphisms (cf. [13], p. 314, [2]) we can define homomorphisms

(1.1) $$J_{R,n}\colon [\Sigma^{p,q}X, GL(n,\boldsymbol{C})]^\tau \to [\Sigma^{p+n,q+n}X, \Sigma^{n,n}]^\tau$$

and $$J_R\colon [\Sigma^{p,q}X, GL(\infty,\boldsymbol{C})]^\tau \to \pi_s^{0,0}(\Sigma^{p,q}X)$$

for $p\geqq 0$ and $q\geqq 1$ where let $\pi_s^{0,0}(\Sigma^{p,q}X)=\lim_{n\to\infty}[\Sigma^{p+n,q+n}X, \Sigma^{n,n}]^\tau$. We now give definitions of $J_{R,n}$ and J_R below. Let $\Omega_d^{n,n}\Sigma^{n,n}$ denote the subspace of $\Omega^{n,n}\Sigma^{n,n}$ consisting of maps of degree d in the usual sense. Let γ be the τ-map of $\Sigma^{n,n}$ induced by the correspondence of $R^{n,n}$ such that $(x_1,\cdots,x_{2n})\mapsto(x_1,\cdots,x_{2n-1},-x_{2n})$. By adding γ to the elements of $\Omega_1^{n,n}\Sigma^{n,n}$ with respect to the loop addition along fixed coordinates of $\Sigma^{n,n}$ we have a τ-map $t\colon \Omega_1^{n,n}\Sigma^{n,n}\to\Omega_0^{n,n}\Sigma^{n,n}$. Then we obtain $J_{R,n}$ by assigning to a base-point-preserving τ-map $f\colon \Sigma^{p,q}X\to GL(n,\boldsymbol{C})$ the adjoint of the composite

$$\Sigma^{p,q}X \xrightarrow{f} GL(n,\boldsymbol{C}) \overset{i}{\subset} \Omega_1^{n,n}\Sigma^{n,n} \xrightarrow{t} \Omega_0^{n,n}\Sigma^{n,n}$$

where i is the canonical inclusion map.

As is easily seen the diagram

$$\begin{array}{ccc} [\Sigma^{p,q}X, GL(n+1,\boldsymbol{C})]^\tau & \xrightarrow{J_{R,n+1}} & [\Sigma^{p+n+1,q+n+1}X, \Sigma^{n+1,n+1}]^\tau \\ \uparrow j_* & & \uparrow \Sigma_*^{1,1} \\ [\Sigma^{p,q}X, GL(n,\boldsymbol{C})]^\tau & \xrightarrow{J_{R,n}} & [\Sigma^{p+n,q+n}X, \Sigma^{n,n}]^\tau \end{array}$$

is commutative under the identification $\Sigma^{r,s}\wedge\Sigma^{p,q}=\Sigma^{r+p,s+q}$ where j_* is the