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ON F-PROJECTIVE STABLE STEMS

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In this note we study F-projective stable stems in dimension n with $7 \leqq n \leqq 22$, where F denotes the complex ($F=C$) or quaternionic ($F=H$) number field. D. Randall [9] determined them in dimension $\leqq 6$.

We use the notations and terminologies defined in the previous paper [8] or the book of Toda [11] without any reference.

1. Definitions and results

Given a pointed space X and a positive integer m, we define

$$\pi_m^{SF}(X) = \begin{cases} \text{image of } p_n^*\colon \{FP_n, X\} \to \{S^{nd-1}, X\} & \text{if } m = nd-1 \\ 0 \quad \text{if } m \not\equiv -1 \bmod (d)\,. \end{cases}$$

An element of $\pi_m^{SF}(X)$ is said to be *F-projective*. In this note we only consider the case of X being the spheres. Remark that $\pi_{nd-1}^{SF}(S^l)$ is a subgroup of G_{nd-l-1}. We say that the m-stem G_m is *fully F-projective* if there exist integers l and n with $m=nd-l-1$ and $\pi_{nd-1}^{SF}(S^l)=G_m$.

Given a positive integer m, we consider the following problems.

(Q.1)$_m$ Compute $\pi_{nd-1}^{SF}(S^l)$ for each n and l with $m=nd-l-1$.

(Q.2)$_m$ What elements of G_m are F-projective?

(Q.3)$_m$ Is G_m fully F-projective?

Of course answers of (Q.1)$_m$ solve (Q.2)$_m$ and (Q.3)$_m$. Our main results are tabled as follows. Here 0 means that the problem is completely solved but no signed place not completely solved yet*). Details are given in (1.6) and § 2.

In what follows in this section we prove some general results. Since p_n^H is the composition of p_{2n}^C and the canonical map $CP_{2n} \to HP_n$, we have

*) Recently in his dissertation, R.E. Snow has determined the C-projectivity of the 2-components for the stems less than or equal to 15.