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# ON STABLE JAMES NUMBERS OF STUNTED COMPLEX OR QUATERNIONIC PROJECTIVE SPACES

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Following James [7] we denote the stunted complex ($F=C$) or quaternionic ($F=H$) projective spaces by $FP_{n+k,k}$ (or $P_{n+k,k}$) for positive integers $n$ and $k$, that is

$$FP_{n+k,k}=FP_{n+k}/FP_n=FP^{n+k-1}/FP^{n-1}\,.$$

Let $d$ be the dimension of $F$ over the real number field. Let $i$: $S^{nd}=FP_{n+1,1}\rightarrow FP_{n+k,k}$ be the inclusion. By *stable James number* $F\{n,k\}$ we mean the order of the cokernel of

$$\deg=i^*\colon \{FP_{n+k,k}, S^{nd}\}\rightarrow\{S^{nd}, S^{nd}\}=Z$$

where $\{X, Y\}$ denotes the group of stable maps from a pointed space $X$ to an other pointed space $Y$. In the previous papers [5, 8, 9, 10] we used the notations $k_s(FP_n^{n+k-1}, S^{nd})$ instead of $F\{n,k\}$ and estimated $F\{1,k\}$.

The first purpose of this note is to determine $F\{n,k\}$ for small $k$, that is, we shall determine $H\{n,k\}$ for $k\leqq 4$, estimate them for $k=5$, determine $C\{n,k\}$ for $k\leqq 8$ and estimate them for $k=9$ and 10. These shall be done in §2 and §3. The second purpose is to show that $F\{n,k\}$ can be identified with the James numbers defined by James in [6]. This shall be done in §4.

An application of this note to $F$-projective stable stems shall be given in [11].

In this note we work in the stable category of pointed spaces and stable maps between them, and we use Toda's notations of stable stems and Toda brackets in [14] freely.

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## 1. Preliminaries

In what follows we shall be working with both real $K$-cohomology theory $KO^*$ and complex $K$-cohomology theory $K^*$. We use the following notations. $KO^*$ and $K^*$ denote both the $K$-functors and the coefficient rings. By the same letter $\xi=\xi_n$ we denote the canonical $F$-line bundle over $FP_n$,