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COHOMOLOGY OPERATIONS IN THE LOOP SPACE OF THE COMPACT EXCEPTIONAL GROUP F_4

Dedicated to Professor A. Komatu on his 70-th birthday

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1. Introduction

Let F_4 be the compact, simply connected, exceptional Lie group of rank 4. In [6] we have described the Hopf algebra structure of $H_*(\Omega F_4;Z)$. Using this thoroughly, we can compute the action of the mod p Steenrod algebra $\mathcal{A}_p$ on $H^*(\Omega F_4;Z_p)$ for every prime p. But here we deal with the cases $p=2$ (Theorem 4) and $p=3$ (Theorem 5) only, because in the other cases the result follows immediately from a spectral sequence argument for the path fibration $\Omega F_4 \to PF_4 \to F_4$.

Let $C(=C_s)=T^1\cdot Sp(3)$ in the notation of [3], which is a closed connected subgroup of F_4. Then in [6] the homogeneous space F_4/C has been found to be a *generating variety* for F_4. That is, there exists a map $f_s: F_4/C \to \Omega F_4$ such that the image of f_{s*}: $H_*(F_4/C; Z) \to H_*(\Omega F_4; Z)$ generates the Pontrjagin ring $H_*(\Omega F_4; Z)$. In this situation Bott [1, §6] asserted that the Steenrod operations in $H^*(\Omega F_4; Z_p)$ can be deduced from their effect on $H^*(F_4/C; Z_p)$. This is the motive of our work.

Throughout the paper X will always denote any connected space such that $H_*(X; Z)$ is of finite type.

2. The generating variety

In this section we shall compute the $\mathcal{A}_p$-module structure of $H^*(F_4/C; Z_p)$ for $p=2$ and 3.

First since C contains a maximal torus T of F_4, we have a commutative diagram

$$(2.1)\qquad \begin{array}{ccc} F_4/T & \xrightarrow{q} & F_4/C \\ \downarrow{\scriptstyle \iota} & & \downarrow{\scriptstyle j} \\ BT & \xrightarrow{\rho} & BC\,. \end{array}$$

We require the following notations and results (2.2)-(2.6), whose details can be found in [3, §4]: