Ikeda, A. and Yamamoto, Y.
Osaka J. Math.
16 (1979), 447-469

ON THE SPECTRA OF 3-DIMENSIONAL LENS SPACES

AKIRA IKEDA AND YOSHIHIKO YAMAMOTO

(Received July 28, 1978)

Introduction. Let (M,g) be a compact connected riemannian manifold and Δ the Laplacian acting on the space of differentiable functions on M. We denote by Spec(M,g) the set of all eigenvalues of Δ;

$$\mathrm{Spec}(M,g)=\{0=\lambda_0<\lambda_1\leqq\lambda_2\leqq\cdots\leqq\lambda_i\leqq\cdots\},$$

where each λ_i is written a number of times equal to its multiplicity. We call it *the spectrum* of (M,g). Two riemannian manifolds (M,g) and (N,h) are said to be *isospectral* to each other if Spec(M,g)=Spec(N,h). What are determined by the spectrum of (M,g)? This problem have been studied by many people; as in Berger [2], Colin de Verdiere [6], Duistermaat-Guillemin [7], MaKean-Singer [8], Sakai [9], Tanno [11] and so on. For example, the spectrum of (M,g) determines the dimension of M, the volume of (M,g) and the lengths of closed geodesics of (M,g) etc.

We are interested in the riemannian manifolds of positive constant curvature, and consider whether they are determined by their spectra. Berger (for $n=2,3$) and Tanno (for $n=4,5,6$) have shown that the standard sphere S^n and the standard real projective space $P^n(\boldsymbol{R})$ are completely characterized by their spectra as riemannian manifolds. The lens spaces are familiar examples of compact riemannian manifold of positive constant curvature. Recently, Tanaka [10] have shown that if a 3-dimensional compact riemannian manifold is isospectral to a lens space with fundamental group of order q, then the manifold is isometric to one of the 3-dimensional lens spaces with fundamental group of order q. In particular a 3-dimensional homogeneous lens space is characterized by its spectrum as a riemannian manifold.

Now, we state our Main Theorem.

Main Theorem. *Let q be a positive integer. If two 3-dimensional lens spaces with fundamental group of order q are isospectral to each other, then they are isometric to each other.*

This theorem will be shown here in this paper only for $q=l^\nu$, $2l^\nu$ and 2^ν where l is an odd prime and $\nu\geqq 1$. In case of any composite number q, the