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BOUNDARY POINTS OF THE DIRICHLET FUNDAMENTAL REGION

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1. Introduction

Recently many of the geometric aspects of the classical theory of Fuchsian groups have been extended by Eberlein and O'Neill [5] and by Eberlein [1]-[4] to a class of simply connected complete riemannian manifolds including those with sectional curvature $K \leqq c < 0$. In [2] and [5] it was shown how to compactify any n-manifold in this class to obtain a topological n-manifold $\bar{M} = M \cup M(\infty)$ homeomorphic to D^n. Since the ideal boundary $M(\infty)$ consists of classes of asymptotic geodesic rays, any group of isometries Γ acting on M extends naturally to a group of homeomorphisms of $\bar{M}$.

In this paper we consider discrete groups Γ of isometries possibly containing elliptic elements acting on manifolds in this class. Given a non fixed point p_0 of Γ, let F_0 denote the open Dirichlet fundamental region for Γ based at p_0 and let $G_0 = cl(F_0)$ where the closure is taken in M. Let $\partial^\infty(F_0)$ denote those points of the cone closure of G_0 in $\bar{M}$ lying in the ideal boundary $M(\infty)$. We first prove (Theorem 3.2) that $x \in \partial^\infty(F_0)$ if and only if all the points of the orbit $\Gamma(p_0)$ lie on or outside the horosphere $L(p_0, x)$ passing through p_0 determined by x. One consequence of this result is that axial fixed points of Γ cannot lie in $\partial^\infty(F_0)$. Thus the union of the cone closures of the translates of the Dirichlet fundamental region need not pave $\bar{M}$.

Motivated by recent results in the theory of Fuchsian groups, we then restrict our attention to geometrically finite discrete isometry groups. Here we will say that Γ is *geometrically finite at* p_0 if the Dirichlet fundamental region F_0 for Γ based at p_0 has only finitely many sides. If Γ is geometrically finite at p_0 and $x \in L(\Gamma) \cap \partial^\infty(F_0)$, then x in fact lies in $\partial^\infty(bd\ F_0)$.

It is a classical result of automorphic function theory ([8], p. 133) that if the fundamental polygon for a Fuchsian group is finite sided, then the boundary points of the fundamental polgyon in S^1 are either ordinary points or para-

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