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A DECOMPOSITION OF THE SPACE $\mathcal{M}$ OF RIEMANNIAN METRICS ON A MANIFOLD

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0. Introduction

Let M be a compact C^∞-manifold. We denote by $\mathcal{M}$, $\mathcal{D}$ and $\mathcal{F}$ the space of all riemannian metrics on M, the diffeomorphism group of M, and the space of all positive functions on M, respectively. Then the group $\mathcal{D}$ and $\mathcal{F}$ acts on $\mathcal{M}$ by pull back and multiplication, respectively. D. Ebin and N. Koiso establish Slice theorem [4, Theorem 2.2] on the action of $\mathcal{D}$.

In this paper, we shall give a decomposition theorem on the action of $\mathcal{F}$ (Theorem 2.5). That is, there is a local diffeomorphism from $\mathcal{F}\times\bar{C}$ into $\mathcal{M}$ where $\bar{C}$ is a subspace of $\mathcal{M}$ of riemannian metrics with volume 1 and of constant scalar curvature τ_g such that $\tau_g=0$ or $\tau_g/(n-1)$ is not an eigenvalue of Δ_g. Combining the above theorems, we get the following decomposition of a deformation (Corollary 2.9). Let $g\in\bar{C}$ and $g(t)$ be a deformation of g. Then there are a curve $f(t)$ in $\mathcal{F}$, a curve $\gamma(t)$ in $\mathcal{D}$ and a curve $\bar{g}(t)$ in $\bar{C}$ such that $\delta\bar{g}'(0)=0$, which satisfy the equation $g(t)=f(t)\gamma(t)^*\bar{g}(t)$. (For the operator δ, see **1.**)

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1. Preliminaries

First, we introduce notation and definitions which will be used throughout this paper. Let M be an n-dimensional, connected and compact C^∞-manifold, and we always assmue $n\geqq 2$. For a vector bundle T over M, we *denote by* $H^r(T)$ the space of all H^r-sections, where H^r means an object which has derivatives defined almost everywhere up to order r and such that each partial derivative is square integrable. Then $H^r(T)$ is isomorphic to a Hilbert space and *the space* $C^\infty(T)$ of all C^∞-sections becomes an inverse limit of $\{H^r(T)\}_{r=1,2,\cdots}$. Therefore such a space is said to be *an ILH-space*. If a topological space $\mathcal{X}$ is isomorphic to an ILH-space locally, $\mathcal{X}$ is said to be *an ILH-manifold*. For details, see [5].