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ON THE SECOND DERIVATIVE OF THE TOTAL SCALAR CURVATURE

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0. Introduction

Let M be a compact connected C^∞-manifold of dimension $n \geqq 2$. It is a classical result due to D. Hilbert [4] that a metric g on M is an Einstein metric if and only if g is critical for the scalar curvature τ, that is, g is such that

$$\frac{d}{dt}\Big|_0 \int \tau_{g(t)} v_{g(t)} = 0$$

for any volume preserving deformation $g(t)$ of g, where $\tau_{g(t)}$ is the scalar cuvature of $g(t)$ and $v_{g(t)}$ is the volume element of $g(t)$. As for the derivative of second order of the integral, Y. Muto [8] shows that there exist volume preserving deformations which gives positive derivative and which gives negative derivative.

In this paper we attempt to decide the sign of the derivative for given volume preserving deformations. The results are as follows. Let (M, g) be an Einstein manifold with certain condition (in Theorem 2.5). If (M, g) is not the standard sphere, then any volume prserving deformation is decomposed to a conformal deformation with positive derivative (Theorem 2.4), a trivial deformation with zero derivative and a deformation of constant scalar curvature with negative derivative (Theorem 2.5).

The paper is organized as follows; after some preliminaries in **1**, we prove the above propositions in **2**. Finally, in **3**, we consider the case when M is a complex manifold and $g(t)$ are Kähler metrics.

1. Preliminaries

First, we introduce notation and definitions which will be used throughout this paper. Let M be an n-dimensional, connected and compact C^∞-manifold, and we always assume $n \geqq 2$. For a riemannian manifold (M, g), we consider the riemannian connection and use the following notation;

v_g; the volume element defined by g,

R; the curvature tensor defined by the riemannian connection,