Sato, H.
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ON MAXIMAL QUOTIENT RINGS OF QF-3 1-GORENSTEIN RINGS WITH ZERO SOCLE

HIDEO SATO

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Until now, there appeared several papers which deal with characterizations for a ring to have a QF (quasi-Frobenius) maximal left quotient ring. Among them, Masaike's one [9, Theorem 2.1] seems to us to be the most decisive in a certain direction. As for QF maximal two-sided quotient rings, we shall give another characterization (Theorem 4.2). Its classical version will be given in Theorem 4.4.

On the other hand, Lenagan investigated noetherian rings with Krull dimension one (see [7] and [8]). A ring R is said to satisfy the restricted minimum condition for left ideals if R/I is an artinian module for any dense left ideal I. Applying Lenagan's result and Theorem 4.4, we shall obtain our main theorem that if a noetherian QF-3 ring with zero socle satisfies the restricted minimum condition for left ideals and right ideals, then it has a QF classical (two-sided) quotient ring (Theorem 5.7). As its corollary, we shall see that a QF-3 1-Gorenstein ring with zero socle has a QF classical quotient ring.

Throughout this paper, we assume that all rings have identity elements and all modules are unitary, and we use the Lambek torsion theory except for §2, unless otherwise specified. We denote by $E(M)$ the injective hull of a module M.

When the author was preparing the present paper, T. Sumioka showed him the paper [16] which appears in this volume. The author would like to express his thanks to Dr. Sumioka for his kindness.

1. Preliminaries

We recall some definitions and results which we need in the sequal. A ring is said to be left 1-Gorenstein if it is left and right noetherian and if it has left self-injective dimension at most one. A left and right 1-Gorenstein ring express his thanks to Dr. Sumi- is called 1-Gorenstein in short.

Let M be a left R-module. When M has Krull dimension, we denote its Krull dimension by K-$dim\ M$ in the present paper. If a ring R is left noether-