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ON GROUPS $H^n(S/R)$ RELATED TO THE AMITSUR COHOMOLOGY AND THE BRAUER GROUP OF COMMUTATIVE RINGS

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The Amitsur cohomology with respect to the unit functor has been studied by many authors. One of the most interesting features of the theory is that its second cohomology group $H^2(S/R, U)$ gives a description of the Brauer group $Br(S/R)$ in far general cases beyond Galois extensions ([1], [13]). But in ring case the extension S/R must satisfy some restrictive condition for the validity of the isomorphism, and Chase and Rosenberg established an exact sequence which is comprised of the unit cohomology, Pic cohomology and the Brauer group, instead of the direct description of $Br(S/R)$ ([4]).

In a preceding paper, we attached a series of abelian groups $\boldsymbol{H}^n(S, G)$ to a commutative ring S and a group G operating on S, which are defined in close connection with the Pic-valued group cohomology, and we showed that if S is a finite Galois extension of R with G as the Galois group, $\boldsymbol{H}^2(S, G)$ is isomorphic to $Br(S/R)$ ([9]), see also [8]).

In this paper, we shall develop a parallel theory in the framework of the Amitsur cohomology, and prove among others that if S is finite projective and faithful as an R-module, our second group is isomorphic to $Br(S/R)$. This extends both the above mentioned case of Galois extensions, and the description by means of the unit-valued cohomology so far established.

In § 1 we shall define the groups $\boldsymbol{H}^n(S/R)$ and prove a long exact sequence which, combined with the interpretation of $\boldsymbol{H}^2(S/R)$ as the Brauer group, yields the Chase-Rosenberg sequence. This part is an immediate transcription of the corresponding part of [9]. The theory of faithfully flat descent precisely fits to the situation around $\boldsymbol{H}^1(S/R)$, and is applied to prove an isomorphism $\boldsymbol{H}^1(S/R) \simeq \mathrm{Pic}(R)$ (§ 2). After some analysis of '2-cocycles' in § 3, we introduce and study a class of algebras denoted by (A, P, p) in § 4. This may be considered as a far more generalized version of the concept of crossed products, and indeed covers the known constructions so far treated in various context. Further, it is immediately observed that the multiplication alteration of Sweedler [15] (hence in particular the construction of Rosenberg and Zelinsky [13] as noted by Sweedler) is nothing but the unit-valued case of our construc-