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SYMMETRIC GROUPOIDS. II

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Introduction

This paper is a sequel to the author's work [8]. It is concerned with the structure of symmetric groupoids (alias "symmetric sets," or "symmetric spaces"), and with the interplay between symmetric groupoids and groups that are generated by involutions (GI groups). As in [8], it is this close relationship between symmetric groupoids and GI groups that provides our *leitsatz*. Recent work on symmetric groupoids by other authors (for instance [2], [4], and [6]) has followed a similar path.

The notation and terminology of [8] will be used without explanation or apology in this paper. Our numbering here begins with Section 5; references to material in Sections 1 through 4 are to the relevant parts of [8]. Nevertheless, the dependence of this work on the earlier one is more apparent than real: the following four sections of this paper can be read with only occasional reference to Sections 1, 2, and 4 in [8].

5. Structure

A few fairly obvious statements can be made concerning the algebraic structure of symmetric groupoids. They will be made in this section.

DEFINITION 5.1. Let A be a symmetric groupoid. A subset B of A is a subgroupoid of A if B is closed under the binary operation of A. If B satisfies

$$a\circ b\in B \quad \text{for all} \quad a\in A \quad \text{and} \quad b\in B,$$

then B is called a *normal subgroupoid* of A

NOTATION. We write $B<A$ if B is a subgroupoid of A, and $B\lhd A$ if B is a normal subgroupoid of A.

Lemma 5.2. *Let A be a symmetric groupoid, and suppose that $B\subseteq A$. Then B is a normal subgroupoid of A if and only if $\xi(b)\in B$ for all $b\in B$ and*

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