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ON A CLASS OF LINEAR EVOLUTION EQUATIONS OF "HYPERBOLIC" TYPE IN REFLEXIVE BANACH SPACES

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1. Introduction

We are concerned with the Cauchy problem for linear evolution equations

$$du/dt+A(t)u=f(t),\ 0\leqslant t\leqslant T, \tag{1.1}$$

of "hyperbolic" type in a Banach space E. "Hyperbolic" type means that the linear opeators $-A(t)$ are the infinitesimal generators of C_0-semigroups on E. In this paper, as T. Kato [1], [2], [3] and [4], we deal with the class that there exists a certain dense linear manifold F in E contained in all the domains $D(A(t))$.

Roughly speaking, our assumptions consist of the reflexivity of E, strong continuity in t of $A(t)$ and its dual $A(t)'$, the stability of $\{A(t)\}$ on E and F (see §2) and the existence of a mollifying operator for $\{A(t)\}$ (see §3). Those are closely related among others to [3]. The main difference lies in weakening the smoothness condition of $A(t)$ in t instead of adding the reflexivity of E. In [3] the norm-continuity of $A(t)$: $F\rightarrow E$ is assumed.

In the proof of our theorem essential use is made of the energy estimates as S. Mizohata [7]. Hence the proof is quite different from [3] in which the integral equations take effect. The author wonders if, even under such a weak smoothness condition of $A(t)$, one can prove *a priori* the strong convergence of $U_n(t, s)$ in §4.

We note here some notations and terminology used in the sequel. The norm of a Banach space E is denoted by $||\cdot||_E$. The inner product by $(\cdot, \cdot)_E$, if E is Hibert. E' is the dual space of E, and $\langle\cdot, \cdot\rangle$ is the scalar product of E' and E. E_w is the locally convex space endowed with the weak topology. Let F be another Banach space. $\mathcal{L}(E; F)$ is the Banach space of all bounded linear operators of E to F with the uniform norm $||\cdot||_{F,E}$, and $\mathcal{L}_s(E; F)$ is the locally convex space with the strong topology. We will abbreviate $\mathcal{L}(E; E)$ as $\mathcal{L}(E)$, $||\cdot||_{E,E}$ as $||\cdot||_E$ and so forth. For a linear operator A of E and a linear manifold $G\subset D(A)$ in E, $A_{|G}$ is the restriction of A to G. A' is the dual of $A\in\mathcal{L}(E; F)$. A^* is the ajoint of A, if A is a densely defined linear operator in a Hilbert