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FORGETFUL SPECTRAL SEQUENCES

Dedicated to Professor A. Komatu on his 70th birthday

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In [1] we descussed general properties of τ-cohomology theories. One of the basic tools in studying a τ-cohomology theory is the forgetful exact sequences which form a natural exact couple. Hence it provides a natural spectral sequence which we call the forgetful spectral sequences. An analysis of this spectral sequence will provide a deeper insight to the structure of the forgetful exact sequence, which we do for $MR^{*,*}(pt)$ in a forth-coming work. These spectral sequences are used partly by Landweber [4] for $MR^{*,*}(pt)$ and by Seymour [9] for $KR^{*,*}(X)$.

In the present work we study basic properties of these spectral sequences. In §1 we study elementary properties of them and show that a forgetful spectral sequence converges to the fixed-point cohomology under certain conditions (Theorem 1.14 and Proposition 1.16). In §2 we see that they have analogies with Bockstein spectral sequences with respect to differentials. In §3 we discuss periodicities which come essentially from Clifford modules. In §4 we study multiplicative properties of them for multiplicative τ-cohomology theories.

1. Definitions and elementary properties

In the present work every τ-cohomology theory is considered on pairs of *finite* τ-complexes for the sake of simplicity. Notations and terminologies of [1] are used freely.

Let $h^{*,*}$ be a τ-cohomology theory. There holds the following exact sequence

$$\cdots \to h^{p-1,q}(X, A) \overset{\chi}{\to} h^{p,q}(X, A) \overset{\psi}{\to} \psi h^{p+q}(X, A) \overset{\delta}{\to} h^{p-1,q+1}(X, A) \to \cdots,$$

called the forgetful exact sequence, for any pair (X, A) of finite τ-complexes [1], (5.1). Set

$$D_1^{p,q} = h^{p,q}(X, A), \quad E_1^{p,q} = \psi h^{p+q}(X, A),$$
$$i_1 = \chi: D_1^{p,q} \to D_1^{p+1,q}, \quad j_1 = \psi: D_1^{p,q} \to E_1^{p,q},$$
$$k_1 = \delta: E_1^{p,q} \to D_1^{p-1,q+1}.$$