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THIN PATCHES AND SEMIPRIME FGC-RINGS

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Throughout this paper all rings are commutative with identity and all modules are unitary. A ring is called *FGC* (*NFGC*) provided every finitely generated (finitely generated nonsingular) module over the ring is a direct sum of cyclic submodules. For a given ring R, we denote the set of all prime ideals of R by spec(R). For a subset X of spec(R), we use $\min X$ and $\max X$ to denote the set of all minimal elements of X and the set of all maximal elements of X, respectively. X is said to be a thin patch if it coincides with the (patch) closure of $\min X$ in spec(R) ([10]).

In this paper, we show the following result, which seems to be a generalization of R. S. Pierce [7, Proposition 20.1]: Let R be a semiprime *NFGC*-ring and X the (patch) closure of minspec(R) in spec(R). Then

(1) $X=\min X \cup \max X$, and
(2) X has no 3-points.

Using this result, we can guarantee the following conjecture*) raised by T. Shores and R. Wiegand ([10]) is indeed true: Every *FGC*-ring has only finitely many minimal prime ideals. Thus, as was point out in [10], we should note that the solution for this conjecture allows us to remove the hypothesis "with Noetherian maximal ideal spectrum" from S. Wiegand [13, Corollary]. Consequently, the structure of a semiprime *FGC*-ring R is completely settled as follows: R is a finite direct product of h-local Bezout domains and each localization of R is an almost maximal valuation ring. The reader is referred to [8]–[12] for the study of *FGC*-rings.

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Let R be a ring. We denote its maximal ring of quotients by $Q(R)$. An R-module is said to be non-singular if every non-zero element of the ring is not annihilated by an essential ideal of R.

For a subset I of R, we put $V(I)=\{x\in\text{spec}(R) \mid x\supseteq I\}$ and $D(I)=\text{spec}(R)-$

*) After writing this paper, I was informed by R. Wiegand that he had already solved this, independently. His proof can be found in [11] or [12].