Ikeda, A. and Taniguchi, Y.
Osaka J. Math.
15 (1978), 515–546

SPECTRA AND EIGENFORMS OF THE LAPLACIAN ON S^n AND $P^n(C)$

AKIRA IKEDA AND YOSHIHARU TANIGUCHI

(Received June 2, 1977)

Introduction

Let M be a compact Riemannian manifold. We consider the Laplace operator Δ acting on the space of differential forms on M. It is a strongly elliptic self-adjoint differential operator, so it has discrete eigenvalues with finite multiplicities. For a given Riemannian manifold, it may be an interesting problem to determine explicitly the spectra and eigenforms of Δ on M. As for the spectra and eigenfunctions of Δ acting on the space of functions, they are known for the cases where M are the following manifolds; flat tori, Klein bottles [3], symmetric spaces [12] and the Hopf manifolds [1]. On the other hand, as for the spectra and eigenforms of Δ acting on the differential forms, we have known no results except for flat tori. But, E. Calabi (unpublished) and recently S. Gallot et D. Meyer [7] have computed the eigenvalues of differential forms on the standard sphere by using the harmonic polynomial forms on $\boldsymbol{R}^{n+1}$.

In this paper, applying the representation theory we compute the eigenvalues of Δ and determine the spaces of eigenforms as representation spaces, on the standard sphere S^n and the complex projective space $P^n(\boldsymbol{C})$ with Fubini-Study metric. Our method is as follows: Let $M=G/K$ be a Riemannian homogeneous space with G acting as transitive isometry group on M. Then Δ is a G-invariant differential operator, so its eigenspaces are G-modules. First, we decompose the space of differential forms on M into G-irreducible modules. In the case where M is S^n, $P^n(\boldsymbol{C})$, or more generally a symmetric space, roughly speaking $\Delta=$ $-$Casimir operator on G. So from Freudenthal's Formula, we can compute the eigenvalues. But the first step of decomposing the space of differential forms on M into G-irreducible modules is generally not easy. In virtue of Frobenius reciprocity law, the problem can be reduced to the following problem: For a given irreducible G-module, how does it decompose into irreducible K-modules? For this problem, a few results are known (cf. H. Boerner [3] and D. P. Zelobenko [15]), and in case $M=S^n$, we apply the known results.

As for the Laplacian Δ acting on the space of functions on S^n and $P^n(\boldsymbol{C})$, its