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H-PROJECTIVE CONNECTIONS AND *H*-PROJECTIVE TRANSFORMATIONS

YASHIRO YOSHIMATSU

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Introduction

Let M be an n-dimensional complex manifold. We write J for its natural almost complex structure. Let ∇ be an almost complex affine connection without torsion on M. A curve $c(t)$ in M is called an *H-planner* curve with respect to ∇ if

$$\nabla_{c'}c' = ac'+bJc' \tag{0.1}$$

for certain smooth functions a and b. Two almost complex affine connections ∇ and ∇' without torsion are said to be *H-projectively equivalent* if they have their H-planner curves in common. From the result of T. Otsuki and Y. Tashiro, this is equivalent to existence of a 1-form ρ on M satisfying

$$\nabla_X Y-\nabla'_X Y = \rho(X)Y+\rho(Y)X-\rho(JX)JY-\rho(JY)JX \tag{0.2}$$

for arbitrary vector fields X and Y ([5], [8]). By an *H-projective* transformation of ∇, we mean a biholomorphic transformation $f: M\to M$ such that $f^*\nabla$ and ∇ are H-projectively equivalent. For example, let $\boldsymbol{P}^n(\boldsymbol{C})=L/L_0$ be the n-dimensional complex projective space of lines in $\boldsymbol{C}^{n+1}$ with the usual connection, where

$$\begin{aligned} L &= SL(n+1, \boldsymbol{C}), \\ L_0 &= \left\{\begin{pmatrix} a & u \\ 0 & B\end{pmatrix}\in SL(n+1, \boldsymbol{C}) \mid B\in GL(n, \boldsymbol{C})\right\}. \end{aligned} \tag{0.3}$$

Then $L/(\text{center})$ is the group of all H-projective transformations.

In the present paper, we shall study H-projective equivalence from the view point of L_0-structure of second order, studied by N. Tanaka and T. Ochiai. In fact, we shall show that H-projective equivalence of ∇ and ∇' is the same as $\boldsymbol{P}^n(\boldsymbol{C})$-equivalence in [6] and [4] (Theorem 1). Therefore, using their results, the family $\{\nabla\}$ of almost complex affine connections without torsion which are H-projectively equivalent to ∇ uniquely determines a Cartan connection ω of type $\boldsymbol{P}^n(\boldsymbol{C})$. This enables us to show that the group of all H-projective