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ON THE IMBEDDING OF DERIVATIONS OF FINITE RANK INTO DERIVATIONS OF INFINITE RANK

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Introduction. Throughout this paper, we shall let $A=k[x_1,\cdots,x_n]$ denote a finitely generated integral domain over a perfect field k. Let p be a maximal ideal of A and set $R=A_p$, the local ring at p. By a k-derivation δ of rank m on R, we shall mean a set $\delta=\{\delta_0,\delta_1,\cdots,\delta_m\}$ of mappings $\delta_i\in\mathrm{Hom}_k(R,R)$ such that δ_0 is the identity map on R and for all $a,b\in R$, $q=1,\cdots,m$, we have

$$(1)\qquad \delta_q(ab)=\sum_{i+j=q}\delta_i(a)\delta_j(b)\,.$$

By a k-derivation D of infinite rank on R, we shall mean an infinite sequence $D=\{D_0,D_1,D_2,\cdots\}$ of k-endomorphisms D_i of R such that for each m, $\{D_0,D_1,\cdots,D_m\}$ is a k-derivation of rank m on R. We shall say that a k-derivation $\delta=\{\delta_0,\delta_1,\cdots,\delta_m\}$ of rank m on R (or A) is integrable on $R(A)$ if there exists a k-derivation $D=\{D_0,D_1,\cdots\}$ of infinite rank on $R(A)$ such that $\delta_i=D_i$, $i=0,1,\cdots,m$.

The problem of finding conditions on R such that every k-derivation of rank m is integrable was to the author's knowlege first suggested by Y. Nakai in [7]. Some work on this problem has been done by several authors. In particular, it follows from [8; (q)p. 33] that if the characteristic of the field k is zero, then every k-derivation of rank m on R is integrable. For this reason, we can assume throughout the rest of this paper that char $k=\rho\neq 0$.

The main results of this paper are the following two theorems: A global results:

Theorem 1. *Let $A=k[x_1,\cdots,x_n]$ be a finitely generated integral domain over a perfect field k. Suppose that for each maximal ideal $p\subset A$, the local ring A_p is regular. Then any k-derivation δ of finite rank on A is integrable on A.*

A complete characterization of regularity on the local level:

Theorem 2. *Let $A=k[x_1,\cdots,x_n]$ be a finitely generated integral domain over a perfect field k. Let p be a maximal ideal of A and set $R=A_p$ (A localized at p). Assume A has dimension r. Then R is a regular local ring if and only if the following two conditions are satisfied:*