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ON CLASSICAL SOLUTIONS IN THE LARGE IN TIME OF TWO-DIMENSIONAL VLASOV'S EQUATION

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1. Introduction

In this paper we study the initial value problem to Vlasov's equation,

$$
(1.1)\quad \begin{cases} \dfrac{\partial f^{\pm}}{\partial t}+\xi\cdot\nabla_x f^{\pm}+\alpha^{\pm}\nabla_x\phi\cdot\nabla_\xi f^{\pm}=0,\ (t,x,\xi)\in[0,\infty)\times R^n\times R^n\,, \\ \Delta_x\phi=\beta\displaystyle\int_{R^n}(f^+(t,x,\xi)-f^-(t,x,\xi))d\xi,\ (t,x)\in[0,\infty)\times R^n\,, \\ f|_{t=0}=f_0(x,\xi),\ (x,\xi)\in R^n\times R^n\,, \\ \nabla_x\phi \text{ is uniformly bounded and } \nabla_x\phi\to 0\ (|x|\to\infty)\,. \end{cases}
$$

Here the unknowns are the functions $f^{\pm}=f^{\pm}(t,x,\xi)$ and $\phi=\phi(t,x)$ where $t\geqq 0$, $x=(x_1,x_2,\cdots,x_n)\in R^n$, $\xi=(\xi_1,\xi_2,\cdots,\xi_n)\in R^n$, and$\nabla_x=(\partial/\partial x_1,\partial/\partial x_2,\cdots,\partial/\partial x_n)$, $\nabla_\xi=(\partial/\partial\xi_1,\partial/\partial\xi_2,\cdots,\partial/\partial\xi_n)$, $\Delta_x=\partial^2/\partial x_1^2+\partial^2/\partial x_2^2+\cdots+\partial^2/\partial x_n^2$, while $\cdot$ denotes the inner product in R^n and $\alpha^{\pm},\beta\in\boldsymbol{R}$. Physically, (1.1) describes the evolution of a rarefied plasma in self-consistent field approximation, where $f^{\pm}$ are respectively the densities of ions $(+)$ and electrons $(-)$ of a plasma at time t in the space of position x and velocity ξ, and ϕ is the potential of electric field of the plasma[1].

If we assume that $f^+\equiv 0$, (1.1) reduces to the initial value problem to the Liouville-Newton equation,

$$
(1.2)\quad \begin{cases} \dfrac{\partial f}{\partial t}+\xi\cdot\nabla_x f+\alpha\nabla_x\phi\cdot\nabla_\xi f=0,\ (t,x,\xi)\in[0,\infty)\times R^n\times R^n\,, \\ \Delta_x\phi=\beta\displaystyle\int_{R^n}f(t,x,\xi)d\xi,\ (t,x)\in[0,\infty)\times R^n\,, \\ f|_{t=0}=f_0(x,\xi),\ (x,\xi)\in R^n\times R^n\,, \\ \nabla_x\phi \text{ is uniformly bounded and } \nabla_x\phi\to 0\ (|x|\to\infty)\,, \end{cases}
$$

where $f=f(t,x,\xi)$, $\phi=\phi(t,x)$, and $\alpha,\beta\in\boldsymbol{R}$. (1.2) is a special case of (1.1), but has an independent physical interest in connection with the dynamics of steller

[1] $\alpha^{\pm}=\mp e/m^{\pm}$, $\beta=-4\pi e$ where e is the unit of electric charge, and $m^{\pm}$ the masses of the ion $(+)$ and the electron $(-)$, and $\alpha=-1$, $\beta=4\pi\gamma m$ where γ is the gravitational constant and m the mass of the particle.