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A CHARACTERIZATION OF BOUNDED KRULL PRIME RINGS

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In [9] we defined the concept of non commutative Krull prime rings from the point of view of localizations and we mainly investigated the ideal theory in bounded Krull prime rings (cf. [9], [10]).

The purpose of this paper is to prove the following:

Theorem. *Let R be a prime Goldie ring with two-sided quotient ring Q. Then R is a bounded Krull prime ring if and only if it satisfies the following conditions;*

(1) *R is a regular maximal order in Q (in the sense of Asano).*

(2) *R satisfies the maximum condition for integral right and left v-ideals.*

(3) *R/P is a prime Goldie ring for any minimal prime ideal P of R.*

As corollary we have

Corollary. *Let R be a noetherian prime ring. If R is a regular maximal order in Q, then it is a bounded Krull prime ring.*

In case R is a commutative domain, the theorem is well known and its proof is easy (cf. [11]). We shall prove the theorem by using properties of one-sided v-ideals and torsion theories.

Throughout this paper let R be a prime Goldie ring, not artinian ring, having identity element 1, and let Q be the two-sided quotient ring of R; Q is a simple and artinian ring. We say that R is an *order* in Q. If R_1 and R_2 are orders in Q, then they are called *equivalent* (in symbol: $R_1 \sim R_2$) if there exist regular elements a_1, b_1, a_2, b_2 of Q such that $a_1R_1b_1 \subseteq R_2$, $a_2R_2b_2 \subseteq R_1$. An order in Q is said to be *maximal* if it is a maximal element in the set of orders which are equivalent to R. A right R-submodule I of Q is called a *right R-ideal* provided I contains a regular element of Q and there is a regular element b of Q such that $bI \subseteq R$. I is called *integral* if $I \subseteq R$. Left R-ideals are defined in a similar way. If I is a right (left) R-ideal of Q, then $O_l(I) = \{x \in Q \mid xI \subseteq I\}$ is an order in Q and is equivalent to R. Similary $O_r(I) = \{x \in Q \mid Ix \subseteq I\}$ is an order in Q and is equivalent to R. They are called a *left order* and a *right order* of I respectively.