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CYLINDRICAL σ-ALGEBRA AND CYLINDRICAL MEASURE

ITARU MITOMA, SUSUMU OKADA AND YOSHIAKI OKAZAKI

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1. Introduction

This paper is devoted to an investigation of Borel structures and measures on locally convex spaces, especially infinite measures. Infinite Borel measure with smoothness was studied in [1]. In Section 2, we refer to some results of [1] (Theorem (**A**), (**B**) and (**C**)) which are used in Section 5.

In Section 3, we examine the relations among the cylindrical σ-algebra $C(X, X')$, the Baire field $B_a(X)$ and the Borel field $B(X)$. For a weakly Lindelöf locally convex space X, we shall show $C(X, X')$ coincides with the weak Baire field $B_a(X_{\sigma(X,X')})$ (Lemma 3.3). Moreover the same result is valid even if X is the strict inductive limit of weakly Lindelöf locally convex spaces (Proposition 3.4). If X is a hereditarily Lindelöf locally convex space, then $C(X, X')$ is identical to $B(X)$ (Theorem 3.6).

In Section 4, we investigate $C(X, X')$-measurability of continuous seminorms. We show if X is a projective limit of separable locally convex spaces, then every continuous seminorm is $C(X, X')$-measurable (Theorem 4.4).

In Section 5, we examine the conditions for a cylinder set measure to be extensible to a pre-Radon or Radon measure. We give a sufficient condition using the results in Section 3 (Proposition 5.1). As an corollary, every totally finite cylindrical measure on X is uniquely extended to a pre-Radon measure on $X_{\sigma(X,X')}$ in case $X_{\sigma(X\ X')}$ has the Lindelöf property. Furthermore if a cylindrical measure μ on X is essentially supported by a $\sigma(X, X')$-Lindelöf subset, then μ is uniquely extensible to a pre-Radon measure on $X_{\sigma(X,X')}$ (Proposition 5.3). In the latter half, we study the Radon extensibility of a cylinder set measure. We present sufficient conditions for the Radon extension of a cylinder set measure (Theorem 5.6). As a corollary of Theorem 5.6, we give another form of Prokhorov's theorem (Corollary 5.7).

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2. Preliminaries

Let X be a set. A family $\mathcal{V}$ of subsets of X is said to be a *paving* if it