

A COMPARISON THEOREM FOR SOLUTIONS OF STOCHASTIC DIFFERENTIAL EQUATIONS AND ITS APPLICATIONS

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Introduction. Comparison theorem for solutions of stochastic differential equations was discussed by A.V. Skorohod [9] and T. Yamada [10]. In §1, we will modify the main theorem of T. Yamada [10] so that it is more convenient for applications. As an application, we discuss in §2 some stochastic optimal control problem which was recently studied by V.E. Beneš [1] using different methods. In §3, we obtain some comparison theorem for one-dimensional projection of a diffusion process. As an example of applications, we see that Hashiminsky's test for explosion ([3], [7]) is obtained simply from a well known one-dimensional result.

1. A comparison theorem for one-dimensional Itô processes

The following theorem is a modification of Theorem 1.1 in T. Yamada [10].

Theorem 1.1. *Suppose we are given the following:*

(i) *a real continuous function $\sigma(t, x)$ defined on $[0, \infty) \times R$ such that*

$$(1.1) \quad |\sigma(t, x) - \sigma(t, y)| \leq \rho(|x - y|), \quad x, y \in R, \quad t \geq 0,$$

where $\rho(\xi)$ is an increasing function on $[0, \infty)$ such that $\rho(0) = 0$ and

$$(1.2) \quad \int_{0+} \rho(\xi)^{-2} d\xi = \infty,$$

(ii) real continuous functions $b_1(t, x)$ and $b_2(t, x)$ defined on $[0, \infty) \times R$ such that

$$(1.3) \quad b_1(t, x) < b_2(t, x), \quad t \geq 0, \quad x \in R.$$

Let $(\Omega, \mathcal{F}, P; \mathcal{F}_t)$ be a complete probability space with right continuous increasing family $(\mathcal{F}_t)_{t \geq 0}$ of sub σ -fields of \mathcal{F} each containing P -null sets and suppose we are given the following stochastic processes defined on it: