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Osaka J. Math.
14 (1977), 609–618

SOME REMARKS ON DEGENERATE CAUCHY PROBLEMS IN GENERAL SPACES

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(Received November 15, 1976)

1. Introduction. We will consider problems of the form

$$u''+s(t)u'+Ar(t)u-A^2a(t)u+b(t)u=f \tag{1.1}$$

$$u(0)=u'(0)=0 \tag{1.2}$$

where A is the generator of a locally equicontinuous group $T(t)$ in a complete separated locally convex space E (cf. [8; 14]), $u\in C^2(E)$, $f\in C^0(E)$, s, r, a, and b are continuous real valued functions, while $a(t)>0$ for $t>0$ with $a(0)=0$. This is an extension of the Cauchy problem for Tricomi equations and various general versions of (1.1)–(1.2) have been considered for example in [1; 2; 7; 8; 10; 15; 16; 18; 22; 23; 24]; for an extensive bibliography see [8]. We will adapt a method of Hersh [13] as extended by the author in [4, 5; 6; 8], to solve (1.1)–(1.2) and prove some uniqueness theorems. The behavior of $\int_\tau^T (r^2/a)(\xi)d\xi$ as $\tau\to 0$ again turns out to play a critical role in uniqueness (as in [7; 8; 23; 24]) and is related to conditions of Krasnov [15] and Protter [18] in their specific contexts. Let us note that a typical case involves $A^2=\Delta$ in a suitable space E (cf. [8]).

2. Following [4; 5; 6; 8; 13] we replace A by $-d/dx$ in (1.1) and consider

$$w''+s(t)w'-r(t)w_x-a(t)w_{xx}+b(t)w=0 \tag{2.1}$$

where $w(t)\in\mathcal{S}_x'$ (detailed properties are indicated below). Let us Fourier transform (2.1) in the x variable, writing formally $\hat{w}(t)=\mathcal{F}w(t)=\int_{-\infty}^{\infty} w(t)\exp ixy\, dx$, to obtain

$$\hat{w}''+s(t)\hat{w}'+iyr(t)\hat{w}+a(t)y^2\hat{w}+b(t)\hat{w}=0 \tag{2.2}$$

It will be convenient to elminate the $b(t)$ term as follows. Let $\hat{w}(t)=\hat{v}(t)\exp\int_0^t\gamma(\xi)d\xi$ where $\gamma(t)$ satisfies the Riccati equation