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DEGREE OF SYMMETRIC KÄHLERIAN SUBMANIFOLDS OF A COMPLEX PROJECTIVE SPACE

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Introduction. Let $P_N(c)$ denote the N-dimensional complex projective space $P_N(\boldsymbol{C})$ endowed with the Fubini-Study metric of constant holomorphic sectional curvature $c>0$. For an irreducible symmetric Kählerian manifold M of compact type, Nakagawa-Takagi [5] constructed a series of full equivariant Kählerian imbeddings

$$f_p: (M, g_p) \to P_{N_p}(c),$$

parametrized by positive integers p, and observed that the degree $d(f_p)$ of f_p (See §1 for the definition) is given by

$$d(f_p) = rp, \quad \text{where} \quad r = \text{rank}\, M,$$

in the case where $p=1$ or M is a complex quadric or a complex Grassmann manifold.

In this note we shall prove the above equality for general symmetric Kählerian submanifolds of $P_N(c)$: Let

$$f_i: (M_i, g_i) \to P_{N_i}(c) \qquad (1 \leqslant i \leqslant s)$$

be the p_i-th full Kählerian imbedding of an irreducible symmetric Kählerian manifold M_i of rank r_i $(1 \leqslant i \leqslant s)$. Take the tensor product (See §2 for the definition)

$$f = f_1 \boxtimes \cdots \boxtimes f_s: (M_1 \times \cdots \times M_s, g_1 \times \cdots \times g_s) \to P_N(c)$$

of the f_i $(1 \leqslant i \leqslant s)$. Then (Theorem 2) the degree $d(f)$ is given by

$$d(f) = \sum_{i=1}^{s} r_i p_i .$$

It should be noted that any full Kählerian immersion f into $P_N(c)$ of a symmetric Kählerian manifold of compact type is obtained in this way.

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