Chahal, J.S.
Osaka J. Math.
14 (1977), 487-500

ARITHMETIC SUBGROUPS OF THE SYMPLECTIC GROUP

JASBIR SINGH CHAHAL

(Received September 27, 1976)

1. Let k be a field and n a positive rational integer. The symplectic group $Sp(n, k)$ of order n over k is the group of $2n\times 2n$ matrices

$$X=\begin{pmatrix} A & B \\ C & D \end{pmatrix} \tag{1}$$

over k, each A, B, C, D being an $n\times n$ matrix, such that

$$X'JX=J\,, \tag{2}$$

where X' denotes the transpose of the matrix X and

$$J=\begin{pmatrix} 0 & E \\ -E & 0 \end{pmatrix},$$

E being $n\times n$ unit matrix. Let $f\colon k^{2n}\times k^{2n}\to k$ be the skew symmetric bilinear form associated with J. Then $Sp(n, k)$ can be identified with the group of automorphisms σ of $2n$-dimensional vector space k^{2n}, such that σ leaves f invariant, i.e.,

$$f(\sigma x, \sigma y)=f(x, y) \tag{3}$$

for all x, y in k^{2n}. It is easy to check that X is in $Sp(n, k)$, if and only if

$$\left.\begin{aligned} &A'C-C'A=0=B'D-D'B \\ &A'D-C'B=E \end{aligned}\right\} \tag{4}$$

and for X in $Sp(n, k)$,

$$X^{-1}=\begin{pmatrix} D' & -B' \\ -C' & A' \end{pmatrix} \tag{5}$$

The group $Sp(n, k)$ is generated by the matrices of the form

$$\begin{pmatrix} E & T \\ 0 & E \end{pmatrix},\ \begin{pmatrix} U & 0 \\ 0 & U'^{-1} \end{pmatrix} \text{ and } \begin{pmatrix} 0 & E \\ -E & 0 \end{pmatrix} \tag{6}$$