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ON THE WEAKLY REGULAR p-BLOCKS WITH RESPECT TO $O_{p'}(G)$

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1. Introduction

We begin with a consequence of a result of Fong ([3] Theorem 1. F.). Let G be a finite group and p a fixed prime number. If D is a defect group of an element of $Irr(O_{p'}(G))$ (that is, D is an S_p-subgroup of the inertia group of an irreducible complex character of $O_{p'}(G)$), then it is also a defect group of a p-block of G. Furthermore, among those p-blocks that have defect group D, there exists a B which is weakly regular with respect to $O_{p'}(G)$. That is, there exists a conjugate class C of G satisfying (1) $C\subset O_{p'}(G)$ (2) C has a defect group D and (3) $\omega_B(\hat{C})\not\equiv 0 \ mod\ \mathfrak{p}$, where $\hat{C}=\sum_{x\in C} x$ (For the definition of the weak regularity, see Brauer [1]).

In this paper, we shall show if D is a defect group of an element of $O_{p'}(G)$, then it is also a defect group of a p-block of G, which is weakly regular with respect to $O_{p'}(G)$. As a corollary, we get if $O_{p'}(G)$ has an element of p-defect d in G, then G has an irreducible character whose degree is divisible by p^{e-d}, where p^e is the p-part of the order of G. As an application of this fact, we shall study those solvable groups all of whose irreducible characters are divisible by p at most to the first power.

NOTATION. p is a fixed prime number. G is a finite group of order $|G|=p^e g'$, $(p, g')=1$. G_p denotes an S_p-subgroup of G. $Irr(G)$ denotes the set of all irreducible characters of G. We fix a prime divisor $\mathfrak{p}$ of p in the ring of integers $\mathfrak{o}=Z[\varepsilon]$, where ε is a primitive $|G|$-th root of unity and we denote by k the residue class field $\mathfrak{o}/\mathfrak{p}$. If C is a conjugate class of G, then we denote by $\hat{C}$ the sum $\sum_{x\in C} x$ in the group ring of G over the field under consideration. Let $F(G)$ denote the Fitting subgroup of G. If G is solvable, we have the normal series,

$$G=F_n \supsetneqq F_{n-1} \supsetneqq \cdots \supsetneqq F_1 \supsetneqq F_0=1, \qquad \text{where} \quad F_i/F_{i-1}=F(G/F_{i-1})\,.$$

The number n is called the nilpotent length of G, which will be denoted by $n(G)$. Some other notations and terminologies which will be used in this paper will be found in Curtis and Reiner [2] or Gorenstein [5].