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ON MULTIPLY TRANSITIVE GROUPS

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1. Introduction

The known 4-fold transitive groups are A_n $(n\geq 6)$, S_n $(n\geq 4)$, M_{11}, M_{12}, M_{23} and M_{24}. Let G be one of these and assume G is a $(4, \mu)$-group on Ω with $\mu\geq 4$. Here we say that G is a (k, μ)-group on Ω if G is k-transitive on Ω and μ is the maximal number of fixed points of involutions in G. Let t be an involution in G with $|F(t)|=\mu$, then $G^{F(t)}=G(F(t))/G_{F(t)}$ is also a 4-fold transitive group. Here we set $F(t)=\{i\in\Omega \mid i^t=i\}$ and denote by $G(F(t))$, $G_{F(t)}$, the global, pointwise stabilizer of $F(t)$ in G, respectively.

In this paper we shall prove the following

Theorem 1. *Let G be a 4-fold transitive group on Ω. Assume that there exists an involution t in G satisfying the following conditions.*

(i) *G is a $(4, \mu)$-group on Ω where $\mu=|F(t)|$.*

(ii) *$G^{F(t)}$ is a known 4-fold transitive group; A_n $(n\geq 6)$, S_n $(n\geq 4)$ or M_n $(n=11, 12, 23$ or $24)$.*

Then G is also one of the known 4-fold transitive groups.

This theorem is a generalization of the Theorem of T. Oyama of [10]: the case that $G^{F(t)}\simeq A_n$ $(n\geq 6)$, S_n $(n\geq 4)$ or M_{12} has been proved by T. Oyama and the case that $G^{F(t)}\simeq M_{11}$, M_{23} or M_{24} by the author.

To consider the case that $G^{F(t)}\simeq M_{23}$ or M_{24}, we shall prove the following theorem in §3 and §4.

Theorem 2. *Let G be a $(1, 23)$-group on Ω. If there exists an involution t such that $|F(t)|=23$ and $G^{F(t)}\simeq M_{23}$. Then we have*

(i) *If P is a Sylow 2-subgroup of $G_{F(t)}$, then P is cyclic of order 2 and $N_G(P)\cap g^{-1}Pg\leq P$ for any $g\in G$.*

(ii) *$|\Omega|=69$ and G is imprimitive on Ω.*

(iii) *$O(G)\neq 1$ and is an elementary abelian 3-group. If we denote by ψ the set of $O(G)$-orbits on Ω, then $|\psi|=23$ and $G^{\psi}\simeq M_{23}$.*

It follows from this theorem that there is no $(3, 24)$-group such that for an involution t fixing exactly twenty-four points $G^{F(t)}\simeq M_{24}$.