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ON THE DEFINING PROPERTIES OF TEICHMÜLLER MAP

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0. Introduction

The concept of Teichmüller mapping seems to be first explicitly introduced by Bers [4] in 1960, significance of which lies, of course, in the fact that it describes a necessary and sufficient condition for a homeomorphism of a closed Riemann surface to be extremal quasiconformal within individual homotopy classes. A few years later the remarkable counter-example was presented by Strebel [14] which showed that the Teichmüller character is no necessary condition for the extremal quasiconformality of mappings between disks with prescribed boundary correspondence: his extremal quasiconformal mapping also plays a part as an example illustrating the non-uniqueness of extremal quasiconformality for the non-compact problem.

Let R, S be a pair of topologically equivalent Riemann surfaces. It does not matter whether they are compact or not. A quasiconformal homeomorphism f of R onto S is customarily called Teichmüller mapping if the Beltrami coefficient μ_f of f satisfies an equation $\mu_f=\kappa\Phi/|\Phi|$ with a positive constant $\kappa(<1)$ and with an analytic differential Φ of type (2, 0) on R at every point of R where $\Phi\neq 0$ (cf. Bers [4], Strebel [14]). Or equivalently, the Teichmüller mapping f is defined as a diffeomorphic solution to the Beltrami differential equation with the coefficient μ_f which equals a constant κ $(0<\kappa<1)$ in modulus and whose argument agrees with the trajectories $\Phi>0$ for some analytic quadratic differential Φ except possibly at its zeros on R.

Here our special attention will be focussed upon the analyticity associated indirectly with Teichmüller mappings. According to Ahlfors [1] it appears to derive from the vanishing of the first variation of maximal dilatation as a functional, so far as the algebraic Riemann surfaces are concerned. On the other hand we know a very simple transcendental example of Teichmüller mapping which is not extremal quasiconformal. What does then characterize the Teichmüller maps at all? The present study arose from an attempt to answer this question, which is also written as a continuation of my previous work [13] in a certain sense. Major part of this paper is devoted to the study on those defining