Saitō, Y.
Osaka J. Math.
14 (1977), 37-53

EIGENFUNCTION EXPANSIONS FOR THE SCHRÖDINGER OPERATORS WITH LONG-RANGE POTENTIALS $Q(y) = O(|y|^{-\varepsilon})$, $\varepsilon>0$

YOSHIMI SAITŌ

(Recieved October 15, 1975)

1. Introduction

The present paper is devoted to developing an eigenfunction expansion theory for the Schrödinger operator

$$S = -\Delta + Q(y) \qquad (y \in \boldsymbol{R}^N) \tag{1.1}$$

with a long-range potential $Q(y)=O(|y|^{-\varepsilon})$, $\varepsilon>0$, as $|y|\to\infty$. This work is a direct continuation of [12] and we shall make use of the results of [12] as main tools throughout this work. Thus, as in [12], in place of the Schrödinger operator S we shall consider the differential operator L with operator-valued coefficients

$$L = -\frac{d^2}{dr^2} + B(r) + C(r) \qquad (r \in I = (0, \infty)) \tag{1.2}$$

with

$$\begin{cases} B(r) = r^{-2}\left(-\Lambda_N + \dfrac{(N-1)(N-3)}{4}\right), \\ C(r) = Q(r\omega)\times \qquad (\omega \in S^{N-1}), \end{cases} \tag{1.3}$$

S^{N-1} being the $(N-1)$-sphere and Λ_N denoting the Laplace-Beltrami operator on S^{N-1}. L can be considered as an operator in $L_2(I, X)$, where $X=L_2(S^{N-1})$ and $L_2(I, X)$ is the Hilbert space of all X-valued functions $f(r)$ on I such that $|f(r)|_X$ is square integrable over I ($|\ |_X$ is the norm of X). Since L is represented as

$$L = USU^{-1} \tag{1.4}$$

by the use of a unitary operator U

$$U: L_2(\boldsymbol{R}^N) \ni F(y) \mapsto r^{(N-1)/2} F(r\omega) \in L_2(I, X) \qquad (r = |y|,\ \omega = y/r \in S^{N-1}) \tag{1.5}$$

from $L_2(\boldsymbol{R}^N)$ onto $L_2(I, X)$, L and S are unitarily equivalent, and hence all the